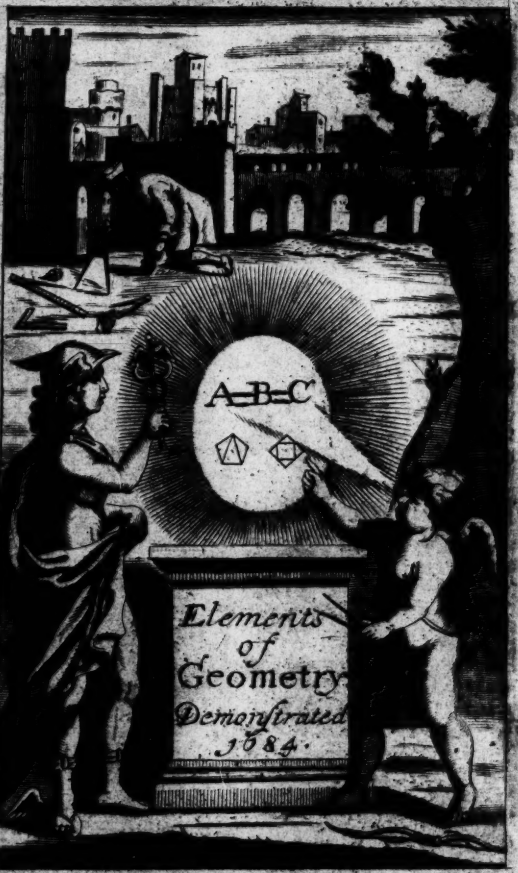
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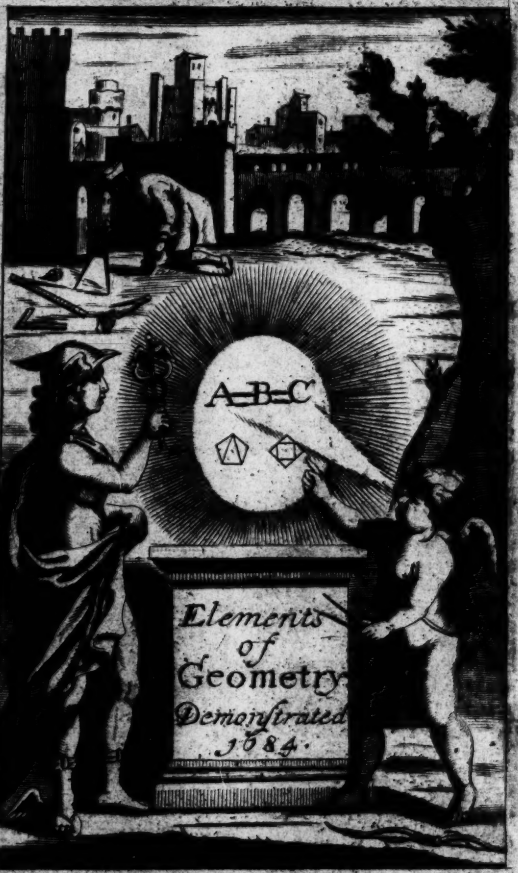
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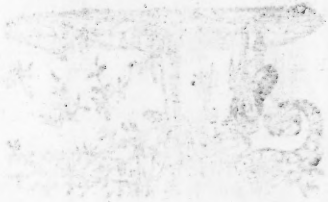
THE ELEMENTS OR PRINCIPLES OF GEOMETRIE.

London, Printed by *J. P.* for *J. Seller* on the *Royal Exchange*, *R. Mount* on *Tower-Hill*, *A. Churchill* near *Amen-Corner*, and *J. Thornton* in the *Minories*. 1684.



INTRODUCTION.

FOR the assistance of those that first enter upon this Study, it may not be amiss to premise some few things by way of Introduction. First then, *Demonstration* is the highest degree of Proof that any matter admits of; fully satisfying the mind in the truth of what is undertaken, and leaving it no further room to doubt. And by consequence nothing is so apt to strengthen our Reason, to give us a clear notion of things, and secure us from being imposed upon by fallacies or shadows of proof, as conversing herein. And of all things that come under our Reason, there is nothing admits of so clear and evident Demonstration as *Geometry*. Where, by taking of matter to pieces we begin with the simplest parts (as they may be call'd) of *Body*, Lines and Points; and clearing our way before us, advance with no less light to the more intricate considerations of its surface and solidity. In all which are discovered variety of delightful properties, that surprise our mind with unexpected truth and conviction, and lay a foundation to many noble and useful Rules of Practice, not onely in the several parts of Mathematics, but in many instances of common life. *Demonstration.*

Demonstration therefore serving (as was said) to enlighten the mind, the Reader must take care in the perusal of these Theorems, not to put himself off with an obscure conception of the matter before him, and a fancy onely that he apprehends it. He must not leave it, till all be as clear as if writ with a beam of the Sun. And tho some of the Theorems may not be of such universal use as others, yet all will be profitable to

perfect the Mind and Reason, when they are thus clearly apprehended.

Directions for reading the following book.

In order to which, the Reader must first endeavour to possess himself of the matter which is to be proved from the *Title* of each Proposition, and then lay down plainly in his mind or paper, how much is supposed, and how much remains to be proved. He must consider also that some things admit of a *direct* proof; as when a thing can be demonstrated that it is so: others onely of an *indirect* proof, when it is demonstrated that it *must be* so; because of some absurdity that would follow from the contrary if it were not so. He must consider that the *Axioms* are truths, which are so naturally clear, that they admit of no further proof but their own light; at least if we duly understand the terms and words in which they are exprest. And to give us an account of the meaning of those terms and words, are the *Definitions* at first laid down, which we must settle well in our memories before we enter upon the book; at least so much of them as relate to that particular part we undertake. To conclude, if any Proposition seems difficult at the first perusal, the Reader is not to be discouraged, or to pore too long upon it, but to pass on to others; which will probably furnish him with light to apprehend what he omitted against a second review.

Of the Axioms.

Definitions.

Before we proceed to give instances of these Directions in some of the following Propositions, it may be necessary to give a few hints to such as are wholly strangers to the Mathematics. 1. That tho in Def. 2. we suppose a Line to be onely an imaginary thing, *a length without breadth*, yet it does not hinder but that we may exprels it

by

by real lines on paper, as we do our thoughts by letters and words. 2. In Def. 6. the length of the sides [a b, a c] Fig. 1. signifies nothing to the greatness of the angle [a]; onely if you extend these sides further from one another, as when you open a two foot Rule, then the angle [a] is thereby increased. 3. When 3 letters are set to express an angle, the middlemost points it out. So if we say the ∠ *bac*, we mean the ∠ *a*. 4. Any side of a △ may be call'd its *Base*, and then the angle opposite to this side is its *Top*. 5. *By (construction)* Construction, we mean the drawing of lines or framing of figures. For the further explication of Def. 37. add this: That is which are of the same kind: for a line and a surface for instance can't be compared together, nor be said one to exceed the other. And of Def. 46. is there meant, that not both the Antecedents and both the Consequents should be in one figure.

Of the 1. Theor.

The I. Theorem, *that one line falling upon another, makes angles equal to 2 ∟*, is immediately founded upon an Axiom, and admits of a *direct* proof. And it is easie for the ingenious Reader to observe from hence, That all the angles that can be made about a point (as *b* Fig. 1.) are equal to 4 ∟. For suppose a line to be run through this point (as *c d*) all the angles above the line will be equal to 2 ∟, according to the I. Theor. And by the same reason all below = to 2 more. And that the Reader is capable to make such a natural inference from any Theorem, is many times supposed in the following Discourse.

Theor. II. explained.

The II. Theorem is brought in upon this occasion. We often suppose (as a thing reasonable to be granted) that a right line already drawn may be continued onward, at either end, by join-

ing another piece to it. But when a piece is so joyned, it is sometimes requisite to be proved that it is rightly joyned, so as (together) to make but one right line. Now this *Proposition* tells us a particular case, in which two pieces so joyned shall certainly be a right line. That suppose *cb* were drawn, and *db* added to it, then *cd* makes but one right line, if a line, as *ab*, falling upon their point of joyning, *b* makes angles with the line *cd* equal to 2 ⌊. Here then is something supposed, and something to be proved. It is supposed that the ∠s above the point *b* are = 2 ⌊, viz. *abc*, *abd*. And it is to be proved that the line *cd* (is well joyned, in *b*, so as to be but) one right line. Now this does not admit of a *direct* proof, onely from an absurdity that would follow if *cd* were not a right line. For if *bd* were not rightly joyned, it must have lean'd either more upward or more downward, (as *br*) there is no third way to be thought on. Put case then, that either of these *cbe* be a right line (and *cd* not one;) this absurdity would follow, that the ∠*abd* would be = to the ∠*abe*; i.e. a part would be = to the whole. For it was supposed (as we have already laid down) that the ∠s *abc*, *abd* were = to 2 ⌊. And if *cbe* be a right line, it follows from the I. Theor. that the ∠s *abc*, *abe* are = to 2 ⌊ also. Wherefore by *Ax.* 6. each pair of ∠s together *abc*, *abd* are = to the other *abc*, *abe* together: since either of these pairs are = to a third thing, namely to 2 ⌊. And leaving out *abc*, which is common to both pairs (according to *Ax.* 8) there remains *abd* = *abe*, which is the absurdity we spake of. So that we can't but be satisfied that *cbd* is well joyned, and makes but one right line. Q.E.D.

(*The thing to be prov'd.*)

This

This may be demonstrated as the foregoing is, from *Ax.*4. by reversing the △, and laying it as it were upon it self. But to avoid the too often repetition of this method, we chose rather to found it upon what we shall here add as a third Sect. to Def. 6. which we shall likewise refer to in Theor. XIX. *Th. XIII.*

Def. 6. Sect. 3. The measure of this inclination is (either an arch of a circle *BC*, of which the angle [*a*] is the centre, which we leave to Trigonometry; or else) a streight line [*gb*]; which being lengthened or shortened (the points *g*, *b* remaining still at the same distances from the angle *a*) will make the inclination of the lines *ab*, *ac* more or less than it is; but if the line *gb* remains the same and in the same place, the inclination will be the same. And by consequence if *IK* be of the same length with *GH*, and the points *I*, *K* at the same distances from the angle *D*, as *G*, *H* are respectively from *A*; then *DI*, *DK* are said to have the same inclination with *AG*, *AH*. *Th. XIX.*

Figures being between the same Parallels are of the same height (Def. 47.) because all Perpendiculars between the same Parallels are =. For this is included in the notion of parallel lines; which are supposed (in the Definition of them) to keep all the way an equal distance from each other. *Th. XXVI.*

The Doctrine of *Proportion*, Chap. III. is somewhat abstracted and nice, and requires a comprehensive mind to take in several things into one thought; wherefore there is the more need of an awakened attentiveness to the perusal of it, and particularly to be well acquainted with the Definitions and Characters belonging to it: but *Proportion.*

but when these once grow familiar, the greatest difficulty is over, and we are let loose into a boundless field of demonstration and variety; for *Proportion* confines not it self to Lines and Figures, but is applicable to Time, and Weight, and Motion, and Force, and in short to whatsoever admits of *greater* and *less*.

Theor. *LXXXIII.* What is there said in the last line, that twice $\frac{5}{6}$ is the same as $1\frac{1}{3}$, may be thus explained: For $\frac{1}{12}$ is $= \frac{5}{6}$, and $\frac{1}{3}$ is $= \frac{5}{2}$, (12 being 4 parts of 16, just as 3 is of 4, and as 9 is of 12.) So that $1\frac{1}{3}$ is $=$ to twice $\frac{5}{6}$.

Th. XCIII. To make the fourth Sect. of this plain, let us call *Twice as many angles as there are sides*, by the name of 2. Now I say that all the inward angles, both at the sides and centre together, are $=$ to B (by 2.) And all the $\angle$s at the sides, both inward and outward together, are $=$ to B (by 3.) Therefore (by *Ax.* 6) all the inward $\angle$s, at the sides and centre together, are $=$ to all the $\angle$s at the sides, both inward and outward, together. Now if from these 2 equal sums you take away the common, viz. all the inward $\angle$s at the sides, there remains (by *Ax.* 8) all the inward $\angle$s at the centre, $=$ to all the outward $\angle$s at the sides. Which $\angle$s at the centre were $=$ to 4$\square$ (by 2.) Therefore the outward $\angle$s at the sides, are $=$ to 4$\square$ also (by *Ax.* 6.) Q.E.D.

From the former part of this Theorem we may frame a Rule, to know the quantity of the angle of any regular Polygon. For since all the $\angle$s together, are $=$ to twice as many $\square$s as there are sides (except 4.) If from the double of the number of the sides we take out 4, and divide the remainder by the number of $\angle$s in the Polygon, the Quotient will give us the measure

of

of each angle. As for instance; In a regular Polygon of 12 sides, the double of the sides is 24; out of which 4 being taken, there remains 20, which (20) divided by 12 gives $1\frac{2}{3}$. So that each angle of such a Polyg. is = to 1 and $\frac{2}{3}$ of a $\llcorner$.

The Doctrine of the *Power of Lines* is the foundation of a great part of *Arithmetic* and *Algebra*; particularly Theor. CV. gives the Rule for extracting the *Square Root*. *Power of Lines.*

This note — between 2 letters, as $A—E$, signifies E subducted from A, and is to be read A less E. ($^{r}=$ in the first line should stand after $\mathcal{AE}$ in the second, so as to be read $\mathcal{AE}$ is $^{r}=$ to $ZE—Eq$. And the reason is because ZE is $= \mathcal{AE}+Eq$ (by CIV.) therefore $\mathcal{AE}$ is $=ZE—Eq$. For if 8 suppose be $=6+2$, then 6 is $=8—2$. It is the same thing whether you take 2 from 8, or add 2 to 6, to make them equal. *Th. CV.*

The conclusion of this Theorem, viz. that $2Q\frac{1}{2}A+E$ are $=2Q\frac{1}{2}A+4\frac{1}{2}\mathcal{AE}+2Eq$. may appear thus (from CV.) For one $Q\frac{1}{2}A+E$ is $=Q\frac{1}{2}A+2\frac{1}{2}A\times E+Eq$. (For in this case $\frac{1}{2}A$ is not look'd upon as a Fraction, but an entire quantity) therefore $2Q\frac{1}{2}A+E$ is = to all the rest taken twice, viz. to $2Q\frac{1}{2}A+4\frac{1}{2}\mathcal{AE}+2Eq$. *Th. CX.*

To conclude, these *Elements* (serving according to the original of their name *eLeMeNts*, as a necessary Key or Alphabet to further progress in Demonstration) are not to be slightly passed over, but imprinted in our minds and memories. And if we go on then to *Trigonometry*, which takes an easie rise from hence, we shall find our selves confirmed more in the use, as well as satisfied in the usefulness of them.

THE

THE

TO THE

INGENUOUS and HOPEFUL

GENTLEMAN,

Mr. RALPH FREMAN,

SON OF

RALPH FREMAN,

OF *ASPEDEN* HALL

ESQUIRE;

THESE

ELEMENTS,

WHICH WERE AT FIRST

PREPARED FOR HIS USE,

ARE NOW WITH DUE RESPECT

PUBLICLY

DEDICATED.

Advertiſement.

THE deſign of this *Treatiſe* being to reduce the *Elements* or *Principles* of *Geometrie* into a ſhort Compaſs, for the benefit of thoſe that haſten to the Practical parts of this Studie, it ſeem'd fit to lay down the following *Definitions*, for the moſt part, as they uſually are expreſt, rather than ſpend time in giving reaſons for the contrary. Though we muſt confeſs with the Ingenuous *Borellio*, that the Angle of the Tangent (or a *mixt* Angle) is indeed no Angle. And perhaps *Geometrie* it ſelf may be better defin'd, the Art of meaſuring *Space*, rather than *Bodies*: And then, a *Surface*

(being the Boundary of such a Space) and Lines, the Boundaries of such Surface, as Points of such Lines, will all naturally appear to be immaterial things. Lastly, the Definition of *Reason* and *Aliquot parts* do not extend to incommensurable Quantities, which are omitted in this short Collection. And some Propositions about Proportion, according to the designed brevity, are rather illustrated, than strictly demonstrated. Those that desire to be curious in these particulars can have recourse to larger Authors. This Summary was thought sufficient for an *Introduction*.

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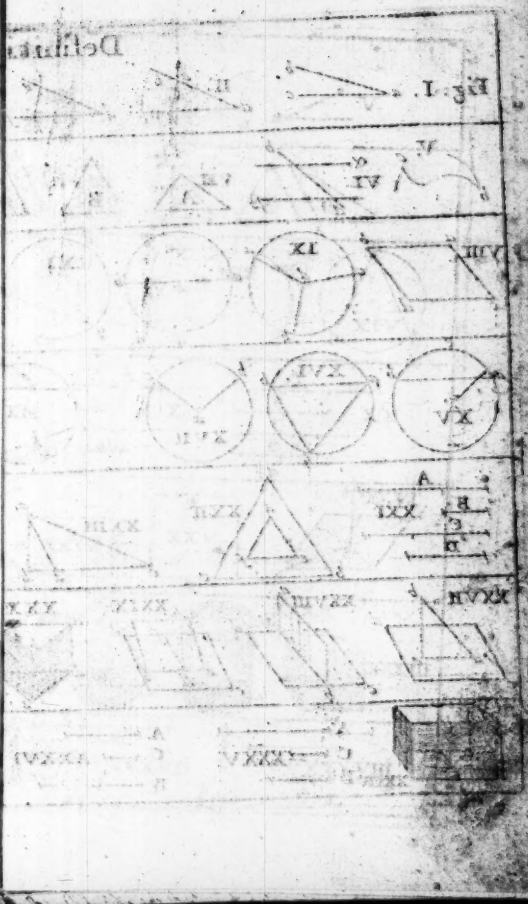

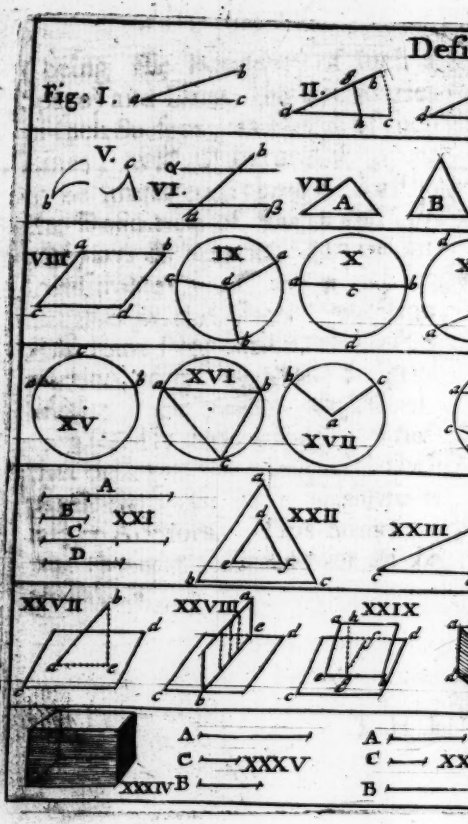
Defi
Fig. I.
II.
V.
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A
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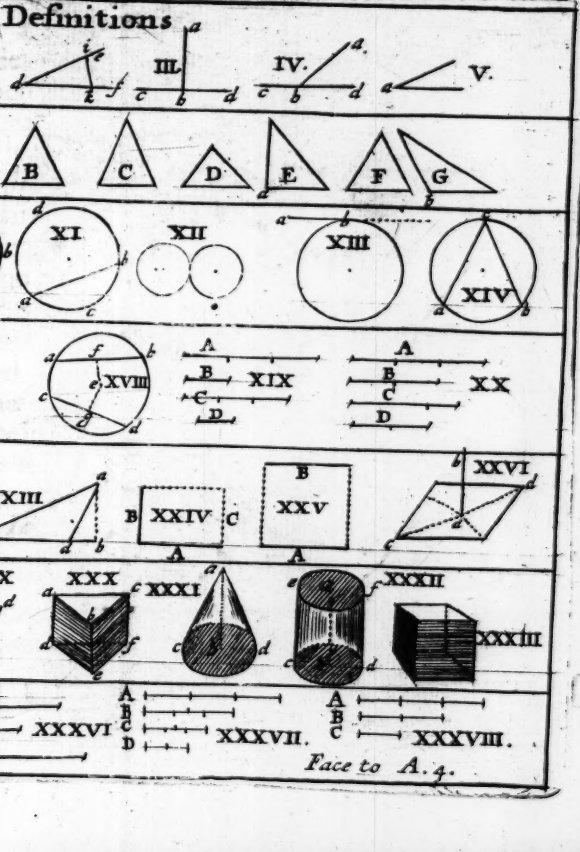
Definitions



Face to A. 4.

THE Elements *or* Principles OF GEOMETRIE.

DEFINITIONS.

CHAP. I.

Of Lines.

GEOMETRIE is the Art of Measuring Bodies: In order to which end, it considers three principal things that belong to a Bodie, namely, Superfice or Surface, Lines, and Points: Which in their proper notion, do signifie, the Out-side, *the* Edg, *and the* Corners *of a Bodie; and so, are real parts of it: But*

in the sense that Geometrie uses them, they do not properly belong to a Bodie, only are applied to it by our imagination. For if a Bodie be dipt in water, though the water touches it every where, yet we may fancy a division between it and the water, which imagmary division, is called a Surface. *And thus, every imaginary division of a Bodie, begets a* Surface; *(between the parts so divided;) of a* Surface, *begets a* Line; *of a* Line *begets a* Point. *And since whatsoever divides a thing, terminates or bounds the parts so divided; therefore,*

1. *A* Point *is said to be bound of a* Line.

2. *A* Line *the bound of a* Surface. *And,*

3. *A* Surface *the bound of a Bodie.*

The first, *having neither length nor breadth: The* second, *length without breadth; and the* third, *length and breadth without depth.*

4. *A* Right Line, *is that which lies ev'n between its two bounding Points, and by consequence, marks out the shortest way from one Point to another.*

5. *A*

5. *A* **Plane Surface** *is that which lies ev'n, between its two bounding Lines, and because it is ſtretcht out ſtrait (like the head of a Drum) therefore it is the ſhorteſt that can lie between the ſame two Lines.*

6. *An* **Angle**, *is the corner* [a] *that is made by the meeting of two Lines:* 1. *Which Angle is greater or leſs, according as the ſaid Lines* [a b, a c] *lean nearer or ſtand further off from one another, which is call'd their* Inclination: 2. *So that if ſeveral Lines* [a b, a c, *and* d e, d f] *have the ſame* Inclination, *the Angles which they make* [a = d] *are equal.* Fig. I. Fig. II.

7. *A* **Right-Angle**, *is when one Line* [a b] *ſo meets another* [c d], *that the two Angles on each ſide,* [a b c, *and* a b d], *are equal to each other.* Fig. III.

Perpendicular: *In this caſe, the Line* A B, *is ſaid to be Perpendicular to* C D.

8. *An* **Obtuſe** *Angle* [a b c] *is that which is bigger than a* Right. Fig. IV.

9. *An* **Acute** *Angle* [a b d] *is that which is leſs than a* Right.

10. *A* **Right-Lin'd Angle** [a] *between two Right-Lines, a* Curv-lin'd, [b] *a* mixt [c]. Fig. V.

11. **Parallel**

Fig. VI. 11. **Parallel** *Lines* [a B] *are those which lean not at all towards one another: So that if they were drawn out infinitely, they would never meet.*

CHAP. II.

Of Figures. PART I.

12. A **Plane Figure**, *is a Plane Surface inclos'd in one or more Lines.*

Fig. VII. 13. *A* **Triangle**, *is a Figure bounded with three Lines*, A.

14. **Equilateral**, *which has all sides equal*, B.

15. **Isoscele**, *or equal legg'd, which has two sides equal*, C.

16. **Scalene**, *which has no sides equal*, D.

17. **Right-Angled**, *which has one Right-angle*, [a] E.

18. **Acute-Angled**, *which has all its Angles Acute*, F.

19. **Obtuse-Angled**; *which has one Obtuse Angle* [b] G.

20. *A*

20. *A* **Parallelogram**, *is a four sided Figure, whose two opposite sides* [a b, c d *or* a c, b d] *are Parallel Lines.* Fig. VIII.

Of a Circle. PART II.

21. A **Circle** *is a plane Figure bounded with one Line, call'd the* Circumference [a b c a], *to which all the Lines* [d a, d c, d b] *that can be drawn from one Point* [d] *in the middle of the Figure, are equal to one another; these Lines are call'd* Radius's, *and the Point in the middle, the* Center. Fig. IX.

22. *The* **Diameter** *of a Circle, is a Right-Line* [a b] *passing through the Center* [c] *bounded at each end with the Circumference, and dividing the Circle into two equal parts.* Fig. X.

23. *A* **Semicircle** *is the Figure* [a d b] *contain'd between the Diameter, and half the Circumference.*

24. *A* **Segment** *(of a Circle) is a Figure contain'd between a Right-Line (call'd a* Chord*) and any part of a Circumference,* [a c b, *or* a d b] *call'd an Arch.* Fig. XI.

25. Equal

25. **Equal Circles**, *are such, whose Diameters or Radius's (that is Semidiameters) are equal.*

Fig. XII. 26. **Circles** *are said to* Touch, *when they do only touch, and not cut one another.*

Fig. XIII. 27. *A* **Right-Line** [a b] *is said to touch a Circle, when being continu'd it does not cut the Circle: This is call'd a* Tangent.

Fig. XIV. 28. *An* **Angle** [c] *is said to* stand upon *that part of a Circumference* [a b] *which is opposit to it.*

Fig. XV. 29. *An* **Angle of the Segment** [a b c] *is made by the Circumference and a Right-Line cutting it.*

Fig XVI. 30. *An* **Angle in the Segment** [c] *is made by two Right-Lines* [a c, b c] *rising from the* Angles of the Segment, *and meeting in the Circumference.*

~~Fig XV.~~ 31. *An* **Angle** *of* Contact, [b] *is between the Tangent and Circumference.* F. 13.

Fig. XVII. 32. *The* **Sector** *of a Circle,* [a b c] *is a Figure made by two Radius's and part of the Circumference.*

Fi. XVIII. 33. **Right-Lines** [a b, c d] *are said to be* Equidistant *from the Center,* [e], *when Lines* [e f, e g] *drawn Perpendicular from the Center to them, are equal.*

CHAP. III.

CHAP. III.

Of Proportion.

34. A **Multiplied** *Magnitude*, [a] *is that which contains another Magnitude*, [b] *a certain number of times preciſely.* Fig. XIX.

35. *An* **Aliquot** *part, or ſimple*; [b] *which being repeated a certain number of times, equals, or meaſures out another* [a] *Magnitude preciſely.*

(1.) Like Aliquot *parts*, [b, d] *are ſuch, as being equally repeated do meaſure out their reſpective wholes*, [a, c.]

(2.) Like **Parts**, *are thoſe that are equally contain'd in their reſpective wholes: Thus* B *and* D *are* like parts, *becauſe* B *is contain'd once and a half in* A, *and* D *once and a half in* C. Fig. XX. Fig. XX.

36. **Ratio** *or* Reaſon, *is the compariſon of two quantities* [a, b] *one with another; whereby, one is ſaid to be bigger or leſs than the other: In which compariſon, that which precedes*, [a] *is call'd the* Antecedent, *and the other* [b] *the Conſequent.* Fig. XIX.

37. *Thoſe*

37. *Those* **Quantities** *only admit of a* Reason, *which being Multiplied may exceed each other.*

Fig. XIX. 38. *The* **Reasons** (*between* A, B, *and* C D) *are said to be the* same (equal, *or* like) *when both the consequents* (B *and* D) *are* like parts *of their respective Antecedents* (A *and* B). *That is, since no quantity can be said to be big or little, but as it is compar'd to another, therefore if* B *and* D *are like parts of* A *and* C, *then* A *is said to be as big, in respect of* B, (*or, to have the* same reason *to* B) *as* C *has in respect of* D: *Thus expreſt,* A . B :: C . D, *or thus,* $\frac{A}{B} = \frac{C}{D}$.

Fig. XXI. 39. *One* **Reason** *is said to be greater or leſs than another, when one of the Conſequents* [b] *is more exceeded by its Antecedent* [a], *than the other* [d] *by its Antecedent* [c]. *That is,* A *is bigger in reſpect of* B, *than* C *is in reſpect of* D, *or, the* Reaſon *of* A *to* B, *is bigger than the reaſon of* C *to* D, *which is thus expreſt;* $\frac{A}{B} \sqsubset \frac{C}{D}$.

40. *The*

40. *The* Equality *of Reasons*, (*mentioned in* def. 38.) *is call'd* Proportion: *That is*, A, B; C, D *are said to be* Proportionals. *And in* def. 39. A, B; C, D *are* Unproportionals.

41. Continual *Proportionals, are when the middle term (or quantity) is taken twice: as when,* A *is as much bigger than* B; *as* B, *is than* C; *which is thus expreſt,* A, B, C, ÷÷ Fig. XXXVII

42. *In the foregoing caſe the reaſon of the firſt term to the third, is ſaid to be* Duplicate *to the reaſon of the firſt to the ſecond; or of the ſecond to the third, (for both theſe reaſons are the ſame, by* def. 38.) *and if there be more terms added, viz.* A, B, C, D, *&c.* ÷÷ *the Reaſon of the firſt* [a] *to the fourth* [d], *is ſaid to be* Triplicate, *&c.*

43. *But if the terms* ABC *are not Proportionals (as in* def. 39.) *then the Reaſon of* A *to* C, *is ſaid to be* compounded *of the Reaſon of* A *to* B, *and of* B *to* C, *thus expreſt,* $\frac{A}{C} = \frac{A}{B} + \frac{B}{C}$. Fig. XXXVIII

44. *The* Homologous *terms in any Proportion, are the two* Antecedents, *or the two* Conſequents: *Thus* A, C, *and* B, D,

D, (*in* Fig. 20.) *are the* Homologous *terms.*

Fig. XXII. 45. *Like* Right-Lin'd *Figures,* [a b c, d e f], *are such as have equal Angles, and the sides about those equal Angles; Proportional, as if the Angle* B, *be supposed to be equal to the Angle* E, *then the side* AB . BC :: DE . EF, *and so of the other Angles and sides.*

46. Reciprocal *Figures, are when you compare the sides of one Figure to the sides of the other, and the Antecedents and Consequents of the Reasons, are in both Figures.*

Fig. XXIII 47. *The* Height *of any Figure, is a Perpendicular Line* [a b] *drawn from the top of it* [a] *to the basis* [c d].

CHAP. IV.

Of the Power of Lines.

Fig. XXIV 48. A Rectangle *is a Parallelogram* (def. 20.) *whose Angles are Right.*

Fig. XXV. 49. *A* Square *is a Rectangle that has all its sides equal, these are also call'd the Powers of Lines.*

CHAP. V.

Of Surfaces.

50. A **Right-Line**, [a b] *is* Right *(or Perpendicular) to a* Plane [c d] (def. 5.) *when all the Lines* [a c, a d, *&c.*] *that can be drawn from that Point* [a] *where it touches the Plain, upon the ſaid Plane do make Right-angles with it,* viz. *when* CAB, DAB, *&c. are Right-angles* Fi. XXVI.

51. *The* **Inclination** *of a Right-Line* [a b] *to a Plane,* [c d] *is meaſured by the Angle* B A C, *where the Line that leans, and the Perpendicular, do touch the Plane.* F. XXVII.

52. *One* **Plane** [a b] *is* Right *to another Plane,* [c d] *when all the Lines in the firſt Plane* [a b] *that are Perpendicular to the common ſection,* [b e] *are alſo Perpendicular to the other Plane* [c d] (def. 50.) F. XXVIII.

53. *The* **Inclination** *of one Plane* [a b] *to another* [c d] *is meaſured by the Angle* [g] *between two Lines* [h g, f g] *in each Plane, which are Perpendicular to the common interſection* [e b]. Fig. XXIX.

54. *This* **Inclination** *will be* equal, (*or* like) *in several Planes when this Angle is equal.*

55. **Parallel** *Planes are such as have no Inclination to one another.*

56. *A* **Solid** *Angle, is a Corner made by the meeting together of several Plane Angles (three at the least) in one Point.*

CHAP. VI.

Of Solids, or Bodies.

57. A **Solid** *or* **Body**, *is that which has Length, Breadth, and Depth.*

58. Like **Solid Figures**, *are such as are contained under an equal number of like plain Figures.*

59. Equal and like **Solid Figures**, *are such as are contain'd under an equal number of like and equal plain Figures.*

60. *A* **Pyramid**, *is a Solid Figure, whose sides are plain Triangles, their several tops meeting together in one Point*

61. *A* **Prism**, *is a Solid Figure, the two opposite sides (or ends)* [a b c, d e f] *of which are* like, *equal, and Parallel; and all the other sides* [a b d e, &c.] *are Parallelograms.* Fig. XXX.

62. *A* **Sphere** *is a Solid Figure bounded with one Surface; to which (Surface) all the ſtrait Lines that can be drawn from one Point within the Figure, call'd the* Center, *will be equal.*

63. *The* **Axis** *of a* Sphere, *is that reſting Right-Line, about which, if a Semicircle be turn'd, it will beget a Sphere.*

64. *The* **Center** *of a Sphere, is the middle of this* Axis.

65. *The* **Diameter** *of a Sphere, is a Right-Line paſſing through the Center, and bounded at each end in the Surface of the Sphere.*

66. *A* **Cone**, *is a Solid Figure, riſing from a Circular baſe* [c d] *according to Right-Lines,* [c a, d a] *and ending in a Point,* [a] *which is call'd the Vertex, or Top.* Fig. XXXI.

67. *The* **Axis** *of a Cone is a Right-Line drawn from the Top,* [a] *to the Center* [b] *of the baſe* [c d].

68. *A* **Right Cone**. *See* def. 71.

Fi. XXXII. 69. *A* **Cylinder** *is a Solid Figure, rising from a Circular base* [c d] (*or Circle*) *according to Right-Lines,* [c e, d f] *and ending in an equal Circle* [e f].

70. *The* **Axis** *of a Cylinder, is a Right-Line,* [a b] *joyning together the Centers of the two Circles.*

71. *A* Right **Cone** *or* **Cylinder**, *is when the Axis is Perpendicular to the base.*

72. Like **Cones** *or* **Cylinders**, *are such, whose Axes, and the Diameters of their bases, are proportional.*

F. XXXIII. 73. *A* **Cube** (*or Dye*) *is a Solid Figure contain'd under six equal Squares.*

74. *A* **Tetraedrum**, *contain'd under Four Triangles, equal and equal sided.*

75. *An* **Octaedrum**, *contain'd under Eight Triangles, equal and equal sided.*

76. *A* **Dodecaedrum**, *contain'd under Twelve Pentagons, equal and equal sided.*

77. *An* **Icosaedrum**, *contain'd under Twenty Triangles, equal and equal sided.*

These five last only, are call'd *Regular* Bodies.

F. XXXIV. 78. **Parallelepipid** (P pp) *is a Solid Figure, contain'd under six Parallelograms, the opposite of which, are Parallel.*

79. *One*

AXIOMS.

79. *One Figure is said to be* Inſcrib'd *in another, when all the Angles of the Figure inſcrib'd, touch either the Angles, Sides or Planes, of the other Figure.*

80. *A* **Figure** *is* Conſcrib'd (*or* Circumſcrib'd) *when either the Angles, Sides or Planes of the outward Figure, touch all the Angles of the Figure that is inſcrib'd.*

Note, That theſe *Definitions*, though put all together, for the convenience of references, will be beſt perus'd ſeverally, before each reſpective *Chap.* in the Book, to which they belong.

AXIOMS.

1. *Between two Points, there cannot lye more than one Right-Line, (nor between two Lines, more than one Right Surface) but they will be* Coincident; *that is, become one. Therefore ſuch Lines (and Surfaces) can't have one common Segment, that is, one part or portion, common to two or more of them.*

2. *All* Right Angles *are equal to one another.*

 3. *Parallel*

Fig. VI

3. *Parallel Lines,* [$\alpha\beta$] (*having no Inclination to one another*) *have the same Inclination to a third Line* [$\gamma\delta$].

4. *Those things which being laid upon one another, do meet in all parts, are equal. The Converse of this* (*to wit, that equals being laid upon one another, will meet*) *is true in Lines and Angles, but not in Figures.*

5. *A* Whole *is greater than any* part *of it; and equal to all its* parts *taken together.*

6. *Things that are equal to a third, are equal to one another: If* A *be equal to* C, *and* B, *be equal to* C, *then* A *is equal to* B.

7. *The* halfs (*or* doubles) *of things that are equal, are equal, and so are any multiples,* &c.

8. *If you add, or take away equal parts from things that are equal, the remainders will be equal.*

9. *If you add, or take away equal parts from things that are* unequal, *the remainders will be* unequal.

AXIOMS.

Of Proportion.

10. *Those things that have the same* Reason *to a third thing, (or to things that are equal,) are themselves equal: That is, those things that are equally great in respect of another, are equal between themselves; and convertedly.*

11. *And if* A, *be greater in respect of* C, *than* B *is in respect of* C, *then* A *is greater than* B. Fi.XXXV.

12. *Or (which is the same in other words)* A *is less than* B, *if the* Reason *of* C *to* B, *be greater* (def. 39.) *than the* Reason *of* A ~~to~~ B: *That is, if* A *be less in respect of* C, *than* B *is, in respect of the same* C, *then* A *is less than* B. F.XXXVI.

13. *Those* Reasons *that are* equal (*or* like) (def. 38.) *to a third* Reason *are equal between themselves, and those that are unequal to a third, are unequal to all that are equal to this third.*

14. *A* Whole, *and all its parts together, have the same* Reason *to a third thing.*

= Equal to.
⊏ Greater.
⊐ Leſs.
+ Added to.
— Subtracted, or divided by.
× Multiplied by.
:: Like.
∴ Continued Proportion.
∟ Right-Angle.
∠ Angle, or Acute-Angle.
△ Triangle.
○ Circle.
□ Square.
▭ Rect-angle.
Perp. Perpendicular.
Pll. Parallel.
Pgr. Parallelogram.
Ppp. Parallelepipedon.
Hyp. Hypotheſis, or Suppoſition.
Pre. Precedent Propoſition.

THEO-

THEOREMS.

[Note. *In these* Theorems *(or Speculative Principles) it is enough to Suppose that Parallel Lines can be drawn, Angles fram'd, and Circles describ'd. The manner how to do them, will be shew'd and Demonstrated in the* Problems *(or Practical Principles) that come after.*]

CHAP. I.

Of Lines. PART I.

Theorem I.

One right Line [a b] *falling upon another* [c d] *makes either two* ∟*s, or such as are* = 2 ∟.

IF A B stands Perp. it makes [a] 2 ∟s ABC, ABD. If it lean, (as EB,) the Angles EBC. EBD, take up the same place as the former 2 ∟ ones had done, and by consequence [b] are equal to them. *Q. E. D.*

[a] *Def.* 7.

[b] *Ax.* 4.

II.

Theorem II.

That [c d] *is a right Line, on which another* [a b] *ſtanding, makes* 2 ∟s *or ſuch as are* = 2 ∟.

IF CD, be not a right Line, then the right Line will fall either under or over it, as CBE. Now the Angles ABC, ABD = [a] (2 ∟ = [b]) ABC, ABE, *i. e.* a part equal to the whole, *Q. E. A.* Therefore C D will be the right Line. *Q. E. D.*

[a] *Sup.*
[b] *Pra.*

Theorem III.

The oppoſite Angles of croſſing Lines (call'd Head *Angles) are equal to one another, as* a = b, *and* c = d.

D + A = [a] (2 ∟ = [b]) D + B, therefore (D being common) A is = B. Alſo A + C = [c] (2 ∟ = [d]) A + D, therefore (A being common) C is = to D. *Q. E. D.*

[a] *I.*
[c] *Ax.* 9.

The reaſon is this, If H F cut E G Perp. all the Angles are ∟ (by I) and therefore equal, (Ax. 2.) ſo then, C = D and A = B. Now if the point O remaining fixt, H F be learn'd aſide, F will come to E at the ſame time that H comes to G, therefore whatſoever is loſt out of the Angles B and A, is equally gain'd by the other

other Angles C and D, so [g] that B will always be equal to A and C = D. Q.E.O. [g] *Ax.* 8.

Theorem IV.

A right Line [$\gamma\delta$] *cutting Parall.* [$\alpha\beta$] *makes all the opposite Angles equal, viz.* a = b = c = d. e = f = g = h

FOr $\alpha\beta$ have [g] the same inclination to $\gamma\delta$, and therefore [h] make = Angles with it. That is, A = C and F = H. But A [i] = B and C [k] = D, also E = F and G = H, and therefore [g] they are all = one another, viz. A = B = C = D, and E = F = G. = H. *Q.E.D.*

[g] *Def.* 11.
[h] *Def.* 6.
[i] *Pre.*
[k] *Ax.* 6.

Theorem V.

And two of the opposite Angles (*whether internal,* [f c. b g] *or external*) [a h, e d] *are equal to* 2 ∟.

FOr, F. A (= [l] C) = [m] 2 ∟ Therefore
B. E (= [l] G) = [m] 2 ∟
F C = B G = 2 ∟. and so of the rest. *Q.E.D.*

[l] *Pre.*
[m] *I.*

VI.

Theorem VI.

If the opposite Angles [abcdefgh] *be equal, the Lines* α β *are* P ll.

n *Sup.*
o *Def.* 6.
p *Ax.* 3.
q *Def.* 11.

SInce the Angle A = n G, the Lines α β have the same o inclination to a third Line γ δ; and therefore p no inclination to one another, that is q to say, are Parallels. *Q. E. D.*

Theorem VII.

Also if the opposite Angles are = 2 L [fc, *or* bg, *or* ah, *or* ed,] *the Lines* α β *are Parallels.*

r *Sup.*
s *I.*
t *Ax.* 8.
u *Pre.*

FOr, FC = r (2 L = s) FA, therefore taking away F which is common, there remains t C = A. Wherefore u α β are Parallels. *Q. E. D.*

Theorem VIII.

Lines Parall. α β *to a third,* δ *are Parall. to one another.*

w *Ax.* 3.

IF you deny it, let α be suppos'd inclin'd to β, therefore w δ is inclin'd to β after the same

same manner, contrary to our supposition, which is absurd; therefore α is not inclin'd to β, and by consequence is Parall. to it. *Q. E. D.*

CHAP. II.

Of Figures. PART. I.

Theorem IX.

In a Triangle [a b c] *the external Angle* [f c b] *is equal to the two internal opposite Angles* [a b.]

LEt $\alpha\beta$ be drawn Parallel to AB. Then the Angle D $=$ [x] A, and E $=$ [x] B. Therefore DE [y] both together, are equal to A $+$ B. *Q. E. D.*

[x] *IV.*
[y] *Ax.* 5.

Theorem X.

The three Angles of every Triangle, are equal to two right Angles.

FOr, CD $=$ [z] 2 $\llcorner$ and D $=$ [a] A $+$ B, therefore CAB $=$ 2 $\llcorner$. *Q. E. D.*

[z] *I.*
[a] *Pre.*

XI.

Theorem XI.

Two sides [a b, b c] *of any Triangle are greater than the third* [a c.]

[b] *Sup.* [c] *Def.* 4.

FOr A C being [b] a right Line, marks [c] out the shortest way between the points A and C, and therefore is shorter than A B C, which lye between the same points. *Q. E D.*

Theorem XII.

In a Triangle [b c d] *equal Angles* [c d] *are subtended by equal sides* [b c, b d.]

[d] *Sup.* [e] *Ax.* 4.

SUppose the whole Triangle to be turn'd the backside forward, (represented by ϱ ↄ ϱ) and laid on the foreside B C D, so that the point ϱ may fall upon C, and ↄ upon D; now because the Angle C = [d] D, therefore [e] the Angle ϱ shall meet exactly with C, and ↄ with D; and by consequence the Line ϱ ϱ shall fall upon C B, and ↄ ϱ upon D B. Lastly, the point ϱ shall fall upon B, (for should it fall any where else, as in E, then the Lines ϱ ϱ and ↄ ϱ would not fall upon C B and D B, against what has been demonstrated) therefore C B = [f] D B. *Q. E. D.*

[f] *Ax.* 1. *and* 4.

XIII.

Theorem XIII.

In a Triangle equal sides [a b, b c] *subtend equal Angles.* [a c]

BEcause the Lines B A = [g] B C, therefore both of them in the point B, are equally distant from the Angles A and C, as are also both of them in their points A and C, (for the line A C is a common measure to both their points;) since therefore A B and B C, are equally distant from the Angles A and C in both their extream Points, and are equal [b] to one another, it follows [i] that the Angles subtended by them, A and C, are also equal. *Q. E. D.*

g *Sup.*
h *Sup.*
i *Def. 6. Cas. 3.*

Theorem XIV.

In a Triangle the greater side [b c] *subtends the greater Angle.* [a]

IF not, let the Angle C be suppos'd = to A, then [k] A B will be = to C B against the supposition. *Q. E. A.* Suppose then C ⊏ A, and take the Angle A C D = A, therefore [l] the side C D = A D and adding D B to both, C D B will be = [l] to A D B. But C D B ⊏ [n] (C B ⊏ [n]) A D B. *Q. E. A. viz.* that the same C D B should both = to and ⊏ than A D B.

k *XIII.*
l *Ax. 8.*
m *XI.*
n *Sup.*

XV.

Theorem XV.

In a Triangle the ⊏ Angle [a] is subtended by the ⊏ side. [b c]

IT is suppos'd that the Angle A is ⊏ than C, and it must be prov'd that the side B C is ⊏ B A; for first, if B C were = B A, then the Angle A would be = ° C, contrary to the Supposition. *Q. E. A.* Or 2*dly*, if B A were ⊏ B C, then the Angle C ? would be ⊏ A, against the Supposition also. Since therefore B C is neither = nor ⊐ B A, it follows that it must be ⊏ B A. Q. E. D.

° *XIII.*

? *Pre.*

Theorem XVI.

In two Triangles if the sides be =, the Angles are =.

FIrst I prove the Angle A = D, for since the sides A B, A C are = to D E, D F, and the subtendent [illegible], it follows ? that the sides, A B, A C, and D E, D F, have the same inclination; and by consequence that the Angles which they contain, A and D, are =. *Q. E. D.*

The same proof will serve for the other Angles.

+ B C = F

? *Def.* 6.

XVI.

Theorem XVII.

In two Triangles, where two sides [b a c *and* e d f] *and one Angle* [a d] *(between those sides) are* =, *all the rest, both sides and Angles, are* =.

SInce the sides B A C and E D F, are =, and also the Angles A and D, therefore being laid upon another, both the sides and Angles shall exactly meet, [r] and by consequence the point E shall fall upon B, and F upon C; therefore [s] the line E F shall exactly meet with B C, and by consequence [t] be = to it; and therefore all are =. *Q. E. D.*

[r] *Ax.* 4.
[s] *Ax.* 11
[t] *Pre.*

Theorem XVIII.

In an equal legg'd Triangle (def. 15) *a line* [b d] *drawn from the top, and cutting the base* [a c] *in the middle* [d], *is perp. to the same base.*

FOr in the two Triangles, A B D and B D C, the side B A = [u] B C, and A D = [u] D C, and B D is common to both, therefore [w] the respective Angles are = to one another; and particularly those two at D, which are therefore [x] ⌊_; and by consequence [y] B D is perp. to A C. *Q. E. D.*

[u] *Sup.*
[w] *XVI.*
[x] *I.*
[y] *Def.* 7.

Theorem XIX.

In two Triangles, where two sides [b a c, e d f] *are* =, *and a third side* [e f,] ⊏ *the Angle* [d] *subtended by this third side, is also* ⊏.

THis follows manifestly from *def.* 6. *cas.* 3. But may be demonstrated (if need were) as XVII, by laying one upon the other.

Theorem XX.

The converse of the former. If two sides [b a c, e d f] *be* = *and one Angle* [d] ⊏, *the subtendent* [e f] *of this Angle, shall be* ⊏ *also.*

IF you deny it, let the subtendent E F be =, or ⊐ than B C, therefore the Angle D will be = or ⊐ than A, contrary to the supposition; wherefore it remains that the subtend. E F should be ⊏ than B C. *Q. E. D.*

Theorem

Theorem XXI.

In two Triangles, if one ſide [a b, α β] *and two Angles* [α, a; β, b] *(adjoyning to this ſide) be* =, *all the reſt ſhall be* =.

SEt αβ upon AB, and they ſhall meet [z]; alſo the Angles α and β being [a] = to the Angles A, and B, ſhall exactly meet [z] with them. Laſtly, The point ∂ ſhall fall upon C (for ſhould it fall any where elſe, as in E, then the ſide β∂ would not fall upon the ſide B C, as has been demonſtrated;) therefore [b] all the ſides are =, and by conſequence [c] the Angles. *Q. E. D.*

[z] *Ax.* 4.
[a] *Sup.*
[b] *Ax.* 1.
[c] *XVI.*

Theorem XXII.

Triangles [a b c, α β γ] *that have two Angles,* [α a, *and* β b,] =, *are equiangular; that is, have all their Angles* =.

WE are to prove that the Angle C = γ. For the three Angles A B C together are = [d] (2 ∟ = [d]) α β γ together. Therefore if from theſe two equal ſumms, you take away A = [e] α and B = [e] β, there will remain [f] C = γ. *Q. E. D.*

[d] *X.*
[e] *Sup.*
[f] *Ax.* 8.

Theorem XXIII.

In two Triangles, where one side, [a b, α β] *and two Angles* [a, α, *and* c, δ] (*though not adjoyning to this side*) *are* =; *all are equal.*

g *X.* THree Angles of S = g (2 ∟ = g) three Angles of O together. If now from these equal summs you take away equals, *viz.* A
h *Sup.* = h α and C = h δ, there will i remain B = β;
i *Ax.* 8. therefore k (the side A B being = h α β) all
k *XXI.* are =. *Q. E. D.*

Theorem XXIV.

In two Triangles [a b c, d e f] *where two sides are* =, *and one Angle* [a = d] (*though not between those sides* (*v. XVII,*) *all are* =; *provided that the other Angles be of the same kind*; viz. ∟, *acute, or obtuse Angle.*

l *Ax.* 4. BEcause the Angle D = A, therefore being
m *Sup.* laid upon it, they will meet, l and the point E will fall on B (because the line D E = m A B,) also the point F, on C: if not, F must fall either below or above, in G or γ. First, not above, for let the Angle F be ∟ or obtuse, then A C B must
n *XIII.* m be the same, which is = n B γ C; (for B γ *i.e.*

E F,

in the △ B γ C

E F, = [m] B C:) therefore A C B and B γ C will be both ⌊ or obtuse, which is contrary to Th. X. 2*dly*, Nor can F fall below, for let F, that is G, be ⌊ or obtuse, it will be = [n] to B C G (because B G (*i.e.* E F) = [m] B C) that is, 2 ⌊ or obtuse Angles in one Triangle B C G, contrary to X. *Lastly*, Let the the Angle F be acute, A C B must [m] be the same, therefore B C G is obtuse = [n] B G C, contr. X. Or (above) let A γ B, *i.e.* F, be acute; therefore B γ C is obtuse = [n] B C γ, contr. X: therefore E F will fall on B C. And the three sides being =, [o] all will be =. *Q. E. D.* [o] *XVI.*

Theorem XXV.

Of several lines that can be drawn from a point given [c], *to a Line*, [a b] *the shortest is a perp.* [c d] *And of the rest, the nearer to this the shorter.*

BEcause in the Triangle C D E the Angle D is ⌊ [p] (and ⊏ [q] then either of the others) therefore the subtendent C E is ⊏ C D. Further, because C E D [q] is acute, therefore [r] C E F is obtuse, and thereupon [s] C F ⊏ C E. *Q. E. D.*

[p] *Sup.* [q] *X.* [r] *I.* [s] *XV.*

Theorem XXVI.

Triangles [c a e, c b e] *upon the ſame baſe* [c e] *and of the ſame height* (*def.* 47) (*or, which is the ſame, between the ſame Parallels* [a b, c d],) *are* =.

DRaw E F and B D Parallel to A C, becauſe the Angle { C E A = t E A F / C A E = t F E A } and
t *IV.*
u *XXI.* the ſide A E is common, therefore u the Triangle C A E = E A F. In like manner the Triangle E F B, = E D B and C D B = C A B. Therefore A C E + E D B (½ of the whole, A C D B) = E D B + C B E; and taking away E D B which is common to both, there remains
w *Ax.* 8. w A C E = C B E. *Q. E. D.*

Theorem XXVII.

Lines [a b, c d] *are Parallels, if equal Triangles* [c a d, c b d] *upon the ſame baſe* [c d] *can ſtand between them.*

IF C D be not parallel to A B, then the Parallel muſt fall either above or below A B, as in F.
x *Pre.* Draw out C B to F. and joyn F D. Therefore
y *Sup.* the triangle C F D = x (C A D = y) C B D. *Q. E. A.*

Theorem

Theorem XXVIII.

Triangles [S X] *upon* = *baſes* [e f. e d], *between the ſame Parallels* [a b. c d], *are* =.

LEt the triangles S and X be ſo plac'd, that A B may be = C F or E D, then joyn A E and B F. Now becauſe C F = A B, and A F is common, and the Angle A F C = [z] F A B, therefore [a] the Triangle S = A F B; in like manner [x] = A E B, and A E B = [b] A F B; ſo then S = (A F B = A E B =) X. *Q. E. D.*

[z] *IV.* [a] *XVII.* [b] *XXVI.*

Theorem XXIX.

Thoſe Lines [a b, c d] *are Parallel, which have between them equal Triangles* [c a e, e b d] *ſtanding on equal baſes.* [c e, e d]

FOr if A E be not Parallel to E D, the Parallel will fall either above or below; which cannot be, as in XXVII; therefore *&c. Q. E. D.*

Theorem XXX.

A Parallelogram [a b c d] *is divided in the middle by the Diameter* [c b] *and the oppoſite ſides are equal.*

[b] *IV.* [d] *XXI.*

BEcauſe the Angle $\left\{\begin{matrix} CBA = [c]BCD \\ BCA = [c]CBD \end{matrix}\right\}$ and the line C B is common, therefore [d] the Triangle C A B = C B D and by conſequence the Pgr. is divided into two = parts. And becauſe of the = Triangles, the ſide A B = C D and A C = B D. *Q. E. D.*

Note, The oppoſite Angles [*a d*, and *c b*] of a Pgr. are =.

[e] *XVI.*

Since the two Triangles A B C and C B D are =, therefore [e] the reſpective Angles are =, *viz.* A = D, and the 2 at C = 2 at B. *Q. E. D.*

Theorem XXXI.

Two Diagonals [a d, c b] *(or Diameters) in a Pgr. cut themſelves in the middle* [e].

[f] *IV.* [g] *XXX.* [h] *XXI.*

BEcauſe in the Triangles A E C and B E D the Angle E B D = [f] E C A, and E D B = [f] E A C, and the ſide A C = [g] B D, therefore [h] the Triangles are =, and by conſequence the ſide A E = E D, and B E = E C. *Q. E. D.*

Theorem

Theorem XXXII.

A Line [a b] *passing by the middle* [e] *of the Diameter* [d c] *cuts the Pgr. into two equal parts.*

BEcause O + Z = [i] X + S, and Z = [k] X, (for the Angle E C A = [l] E D B, and F A C = [l] E D B, and the side E C = [m] E D) therefore [n] taking away the equals Z and X, there remains O = S, which equals being added, it will be O X = S Z. *Q. E. D.*

[i] *XXX.* [k] *XXIII.* [l] *IV.* [m] *up.* [n] *Ax.* 8.

Theorem XXXIII.

In a Pgr. the Complements (*so S and Z are call'd*) *are* =.

FOr O Z A = [o] X S V, all together. Also, O = [o] X and A = [o] V, therefore [p] there remains Z = S. *Q. E. D.*

[o] *XXX.* [p] *Ax.* 8.

Theorem XXXIV.

Pgrs. [a d, c e, f g] *between the same Parallels* [a f, c h] *upon the same* [c d], *or equal* [c d, g h] *bases, are equal.*

WE are to prove that A D = C E = F G, thus, The Triangle C B D (½ [q] A D and of

[q] *XXX.*

q XXVIII. c Ax. 7. of CE) = q EGH (½ of FG.) Therefore r the wholes, AD, CE and FG are, also equal. *Q. E. D.*

CHAP. II.

Of a Circle. PART. II.

Theorem XXXV.

A Line [b d] *passing the Center, is perpendicular to a Chord* [a c] *which it divides in the middle* [d].

t Radius's Def. 21. s Sup. t XVI. u I.

FOr the side EA = t EC } and DE is com-
DA = s DC }
mon, therefore t the Angles at D are =, and by consequence u ⌊. *Q. E. D.* (*Def.* 7.)

Theorem XXXVI.

And if it [b d] *be perpendicular, it divides the Chord* [a c] *in the middle.*

v Radius's. x Sup. y XXIV.

FOr the side EA = v EC, ED is common, and the Angles at D are = x, therefore AD = y DC. *Q. E. D.*

Theorem

Theorem XXXVII.

If it [b d] *divides a Chord* [a c] *in the middle, and be alſo Perpendicular to it, it paſſes through the Center.*

IF not, let E D paſs thongh the Center, therefore the Angle E D C [z] is ∟ = [a] B D C that is, a part = to the whole. *Q. E. A.*

[z] *XXXV.*
[a] *Sup.*

Theorem XXXVIII.

That point [a] *is the Center of a Circle, from whence more than two equal* [a b, a d, a c] *right Lines can be drawn to the Circumference.*

DRaw the Lines B D, D C divided in the middle by A E, A F, becauſe the ſides B E = E D, A B = A D and A E is common, therefore the Angles at E are =, and by conſequence [d] ∟; therefore [e] E A paſſes the Center. As in like manner, by the ſame reaſon does F A. Since therefore both theſe paſs the Center, It muſt be where they meet; *viz.* in A. *Q. E. D.*

[b] *Conſtruct.*
[c] *Sup.*
[d] *I.*
[e] *Pre.*

Theorem

Theorem XXXIX.

Croſſing Chords [a c, b d] (*not paſſing the Center*) *do not cut each other in the middle.*

FOr if E were the middle of both, then F E (paſſing the Center) would make F E C ⌊ [f] to A C, and F E D ⌊ [f] to B D; that is F E C = [g] F E D a part = to the whole. *Q. E. A.*

[f] *XXXV.*
[g] *Ax.* 2.

Theorem XL.

Of ſeveral Lines [a c, a d, a e] *drawn from one point* [a] (*in a Circle*) *to the circumference, the greateſt* [a c] *paſſes the Center; the reſt are* ⊏ *as nearer to this.*

1. B C = [h] B D, therefore A B C (= [i] A B D) ⊏ [k] A D. *Q. E. D.*

2. B F D ⊏ [k] B D (= [h] B F E, and omitting the common B F) F D ⊏ [i] F E; and (adding the common A F) A D ⊏ [i] (A F E [k] ⊏) A E. *Q. E. D.*

[h] *Radius's.*
[i] *Ax.* 8.
[k] *XI.*

Theorem

Theorem XLI.

Of ſeveral Lines [a b, a d, a e] *drawn from one one point* [a] *(without a Circle) to the inner circumference* [b d e] *the greateſt* [a b] *paſſes the Center; the reſt are greater as nearer.*

1. A C + (C D = [l]) C B ⊏ [m] A D. *Q. E. D.*

2. C F D ⊏ [m] (C D = [l]) C F E; therefore (omitting the common C F) F D ⊏ [n] F E, and (adding the common F A) D F A ⊏ [n] (E F A ⊏ [m]) E A. *Q. E. D.*

l *Radius's.*
m *XI.*
n *Ax.* 8.

Theorem XLII.

Of ſeveral Lines [a b, a d, a e] *drawn from one point* [a] *(without the Circle) to the outward circumference,* [b d e] *the leaſt* [a b] *is that which being continued would paſs the Center, the reſt are leſs as nearer to this.*

1. C D A ⊏ [o] C B A, therefore (omitting the equals [p] C D, C B) D A ⊏ [q] B A. *Q. E. D.*

o *XI.*
p *Radius's.*
q *Ax.* 8.

2. Draw

o *XI.* 2. Draw out C D to F, C E F ⊏ o C D F, and (omitting the equals p C E, C D) E F ⊏ q D F, to which adding the common F A, E F A ⊏ q (D F A ⊏ o D A. *Q. E. D.*

p *Radius's.*

q *Ax.* 8.

Theorem XLIII.

If one Circle touches another, a line [b a] *drawn through both their Centers* [b f] *will fall upon the point of touching* [a].

IF not, let the Center of the lesser O be C, and the line that passes through both the Centers B, C, fall on D. Since then C is the center of the ⊐ O, C A = r C E, to which if you add the the common B C, B C E (= s B C A) ⊏ t (B A = u) B D. *Q. E. A.*

r *Radius.*

s *Ax.* 8.

t *XI.*

u *Radius's of the* ⊏ O.

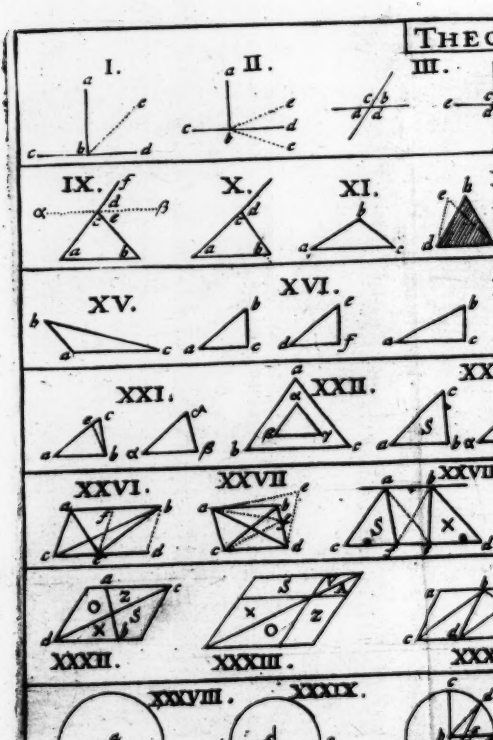
Theorem XLIV.

If Circles touch without, a line [b c] *that joins their centers will pass through the point of touching* [o.]

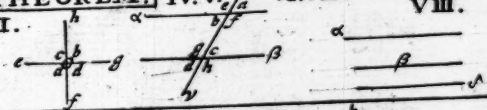
IF O be not the point of touching, let it be A, then B A, C A will be Radius's of their respective Circles; and by consequence = to B O, O C ⊐ w B A, A C, which is a contradiction.

o *XI.*

Theorem

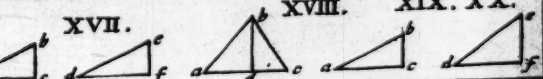

THEO
I.
II.
III.
IX.
X.
XI.
XV.
XVI.
XXI.
XXII.
XXVI.
XXVII
XXXII.
XXXIII.
XXXVIII.
XXXIX.

THEOREM. IV. V. VI. VII. VIII.

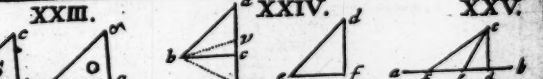


XII. XIII. XIV.

XVII. XVIII. XIX. XX.



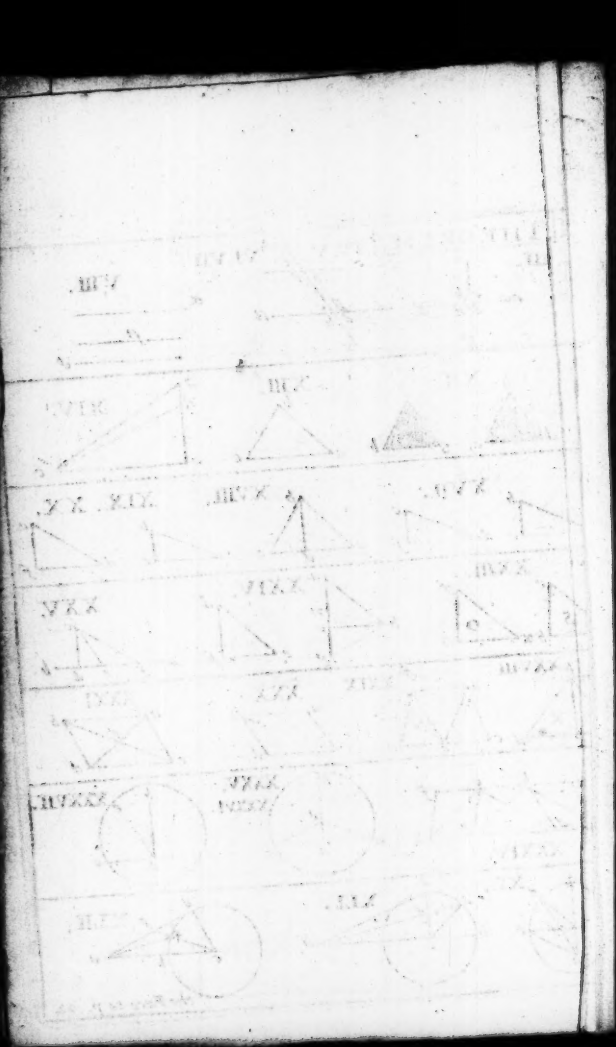
XXIII. XXIV. XXV.



XXVIII. XXIX XXX XXXI

XXXIV. XXXV. XXXVI. XXXVII.

XL. XLI. XLII.



Theorem XLV.

Circles whether within or without, touch but in one point [a].

IF it be poſſible, let them touch alſo in another point, as B, draw the Line E A through the Centers E F; therefore F A = [x] F B, and (adding the common E F) E F A (= [y] E F B) ⊏ [z] E B, againſt *def.* 21.

Alſo if they touch without (as in *fig.* XLIV) in O and A, then B A, A C = B O, O C, contrary to *XI*; wherefore they touch but in one point. *Q. E. D.*

[x] *Radius's.*
[y] *Ax.* 8.
[z] *XI.*

Theorem XLVI.

One Circle cuts another in two points only.

IF it be poſſible let them cut in four, A B C D, therefore four Lines drawn from theſe points to the Center of the O *α*, will be equal [a]. Whereas by *XLII*, thoſe will be ⊏ which are neareſt the Center of the O *β*. If the Centers fall within both Circles, it will be in like manner contrary to *XLI*.

[a] *Radius.*

Theorem

Theorem XLVII.

A perpendicular [b c] *to the end of the Diameter* [b,] *falls all of it without the Circle.*

FRom the Center D, draw the Secant D C, the Angle D B C is ∟ [b], and thereupon ⊏ [c] D C B; therefore the ſubtendent D C ⊏ [d] D B a Radius, and by conſequence the point C is without the Circle. The ſame reaſon will hold for all points between C and B, therefore &c. *Q. E. D.*

[b] *Sup.* [c] *X.* [d] *XV.*

Theorem XLVIII.

The Angle [d a b] *of the Radius* [d a] *and Circumference is* ⊏ *than any acute Angle* [d a c.]

DRaw D E perpendicular to A B; the Angle D E A is ∟ [e], and ⊏ [f] D A E. Therefore the ſubtend. D A ⊏ [g] D E, ſo that D E muſt fall within the O, and by conſequence the Angle D A E is but a part of D A B, and ſo ⊐ than it. *Q. E. D.*

[e] *Conſtr.* [f] *X.* [g] *XV.*

Theorem

Theorem XLIX.

The Tangent [b a] *of a Circle (def.* 27.) *makes a* ⌊ *with the Radius* [c a].

FOr if it made an acute, it would fall within the Circle (h since the Angle of the Circumference and Radius is ⌐ than any acute,) (*against* def. 27.) if an obtuse, then the Angle on the other side C A D, would be acute; and by h consequence wou'd fall within the Circle, against def. 27. h *Pre.*

Theorem L.

Two Tangents [a b, a c,] *drawn from the same Point* [a], *are equal.*

FOr the Angles, D C A and D B A are ⌊ i, and thereupon k =. Also D C B = l D B C, (for D B m = D C,) which being taken from the two foresaid ⌊ Angles; there n remains A B C = A C B, and by o consequence the subtendent A B = A C. *Q. E. D.*

i *Pre.*
k *Ax.* 2.
l *XIII.*
m *Radius.*
n *Ax.* 8.
o *XII.*

Theorem LI.

p *Def.* 33. *Lines*, [ad, bc] p *equidistant from the Center* [e] *are equal.*

LEt fall the Perpendiculars EF, EG, from the Center E.

q *Radius's.* The side $\left\{\begin{matrix} EA^{q} = EB \\ EG^{p} = EF \end{matrix}\right\}$ and the Angle EGA,
s *Constr.* (∟ s) = s EFB. t Therefore the Triangle
s *Ax.* 2. AEG = BEF, and the side AG = BF. In
t *XXIV.* like manner the side GD = FC: therefore the
u *Ax.* 7. whole AD u = BC.

Theorem LII.

Of Lines [ab, ac, ad,] *in a Circle, the greatest is the Diameter* [ad,] *the rest are greater as nearer to this.*

w *AD, AC, being Rad. and AE, common.* 1. AED (w = AEC) ⊏ AC. *Q.E.D.*

2. The Angle AEC ⊏ x AEB, therefore the subtend AC ⊏ y AB, (for the two sides AEC = AEB, all Radius's) *Q.E.D.*

x *Ax.* 5. If it be said that FG, is further from the Center than AC, and yet not ⊐ AC, from the
y *XX.* point A draw a Line (as AB) equidistant from
z *Pre.* the Center with FG, which shall be = z to FG, but ⊐ than AC, as before,

Theorem

Theorem LIII.

The opposite Angles [b d, a c,], *of a four sided Figure* [a b d c] *inscrib'd in a Circle, are equal to* 2 ∟.

DRaw the Radius's, LA, LB, LC, LD; the several Angles subtended by these Radius's will be [a] =, *viz.*

E, F, I, K = G, H, M, N

Now all these Eight together, are = 4 ∟ (b because the Figure A B C D, may be divided into two Triangles) therefore either half of them, E F I K, that is B D; or G H M N, that is, A C, are = 2 ∟. Q. E. D.

[a] *XIII.*

[b] *X.*

Part II.

In case the four sided Figure inscribed, (*def.* 90.) *falls without the Center* [i].

DRaw the Radius, I A, I C, I B, I H, to the several Angles. Now in the Triangle A I H, the Angle A = H; and these Angles also are =, *viz.*

[c] *XIII.*

[c] *XIII.* [d] *X.*

$$\left.\begin{matrix}A\\+\\B\\E\\F\end{matrix}\right\}^{c}=\left\{\begin{matrix}C\\D\\G\\+\\H\end{matrix}\right.$$

Omitting therefore the equals A and H, there remain equal halvs, BEF, and DGH, which altogether, are $= 4\llcorner$ ([d] because the whole Figure BCFG, may be divided into 2 Triangles,) and therefore either half, (being the opposite Angles,) are $=$ to $2\llcorner$. *Q. E. D.*

Theorem LIV.

All Angles [a, e] *in the same Segment,* [b f d] (*def.* 30.) *are equal*

[e] *Pre.* [f] *Ax.* 8.

$$\left.\begin{matrix}C+A=\\C+E^{e}=\end{matrix}\right\}2\llcorner.$$

Therefore (omitting the common C) $A^{f}=E$. *Q. E. D.*

Theorem LV.

An Angle [c f d] *in a Segment* [c e d] *at the Center* [f], *is double to that* [c b d] *which is at the Circumference.*

[g] *IX.* [h] *XIII.* [i] *Radius's.*

FOr the Angle CFD[g] $=$ FDB $+$ B; which FDB[h] $=$ B, (the subtend, FD being[i] $=$ FB) and B $=$ A: Therefore CFD is double to B or A. *Q. E. D.*

Theorem

Theorem LVI.

The Angle [a b c] *in the Semicircle is* ∟.

THe Angles A, B, F + E, D, C [k] = 4 ∟. But F + D [l] = 2 ∟: Therefore A, B, E, C [m] = 2 ∟. But B, E, [n] = A, C, that is, either of these pair are = 2 ∟, and by consequence B + E, the Angle at the Semicircle is ∟. *Q. E. D.*

k *X.*
l *I.*
m *Ax.* 8.
n *XIII.*

Theorem LVII.

An Angle in a Segment { ⊏ / ⊐ } *the Semicircle is* { *Acute*, / *Obtuse*. }

BEcause CBD is [o] ∟, therefore CBA is [p] obtuse; and CBE, [p] acute. *Q. E. D.*

o *Pre.*
p *Def.* 8, 9.

Theorem LVIII.

Equal Angles, [b, e; k, l;] *whether at the Center or Circumference, are subtended by equal Arches,* [a c, d f.] *of* = *Circles.*

BEcause the Angle E [q] = B, and the sides DE F [r] = ABC, therefore being laid upon

q *Sup.*
r *Radius's of* = ○s.

one another, they shall [f] meet, and by consequence the points D, F with the points A, C; likewise the ◯s being [q] =, (if the Center E, be laid upon the Center B,) they shall meet, and by consequence the Arch D F with A C; wherefore they are =. *Q. E. D.*

Now K, L, are subtended by the same Arches; which are already prov'd =.

f *Ax.* 4.

q *Sup.*

Theorem LIX.

The Angle [a e f] *at the Center, standing upon half the Arch* [a f], *is* = *to the Angle at the Circumference, standing upon the whole Arch* [a f e].

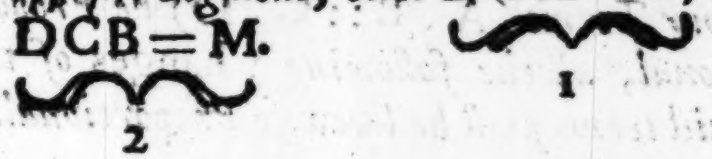
FOr the Angle A E F [t] = C E F, (the Arch A F being [u] = F C) but A E F is [w] double to A D F, and C E F is [w] double to C D F: Therefore A D F [x] = C D F, and both together (*viz.* A D C) are = A E F; but A D C [y] = A B C, therefore [z] A B C = A E F. *Q. E. D.*

t *Pre.*
u *Sup.*
w *LV.*
x *Ax.* 7.
y *LIV.*
z *Ax.* 3.

Theorem

Theorem LX.

A Line [d c] *passing through a Circle at the Point* [c] *of touching, makes an Angle to the Tangent* [a b] = *to an Angle in the opposite Segment, viz.* DCA = E, *and* DCB = M.



DRaw the Diameter F C, and joyn F D.

1. Because the Angles [a] FDC, and [b] FCA, are ∟. Therefore K [c] + H, or D C A + H [d] = ∟, and (omitting the common H) there remains D C A [e] = (K [f] =) E. *Q. E. D.*

2. The Angle E + M [g] = (2 ∟) [h] = D C A + D C B: Therefore (taking away equals, D C A [i] and E,) there remains D C B = M. *Q. E. D.*

[a] *LVI.*
[b] *XLIX.*
[c] *X. & Ax. 5.*
[e] *Ax. 8.*
[f] *LIV.*
[g] *LIII.*
[h] *L. & 62.*

CHAP. III. PART I.

Of Proportion in General.

Theorem LXI.

If four Terms (A . B :: C . D) *be Proportional, all the following* Changes *of the ſaid terms ſhall be likewiſe Proportional.*

	A . B :: C . D	Let { A . B :: C . D 12. 9 :: 8. 6
1. *Inverted.*	B . A :: D . C	9. 12 :: 6. 8
2. *Alterned.*	A . C :: B . D	12. 8 :: 9. 6
3. *Compounded.*	A+B . B :: C+D . D	21. 9 :: 14. 6
Or.	A+C . C :: B+D . D	20. 8 :: 15. 6
4. *Divided.*	A−B . B :: C−D . D	3. 9 :: 2. 6
Or.	A−C . C :: B−D . D	4. 8 :: 3. 6
5. *Converted.*	A . A ± B :: C . C ± D	12. 21 :: 8. 14 12. 3 :: 8. 2
Or.	A . A ± C :: B . B ± D	12. 20 :: 9. 15 12. 4 :: 9. 3
6. *Mixed.*	A+B . A−B :: C+D . C−D	21. 3 :: 14. 2
Or.	A+C . A−C :: B+D . B−D	20. 4 :: 15. 3

That

That all these Changes are Proportional, may appear sufficiently plain from the numbers annext according to the following instances, in Theor. LXII. only Note by the way, that all these Changes of the Letters, agree also to the sides of the Triangle, (Fig. 61.) which it would not be amiss to look upon all the way, as you read these over. *Note* 1.

2. The first Change, *Inverted*, affirms no more than this, that if A be as big in respect of B, as C is in respect of D, then B is as little in respect of A, as D is in respect of C; which may pass for an Axiom. 2.

Theorem LXII.

The second Change, Alterned { A.C :: B.D / 12.8 :: 9.6.

SUppose A, B, C, D; to be Lines divided into so many parts, *viz.* 12, 9, 8, 6; then 12. 8 :: 9 . 6, for 8 is contained once and a ⅓ (*viz.* 4.) in 12, and 6 once and a ⅓ (*viz.* 3.) in 9. So that B and D are like parts of A and C, and by consequence A . C :: B . D. *Q. E. D.*

The same method will make all the rest of the Changes appear proportional.

A ——— B — C ——— D — a —.—.—. b —.

But in order to a further Demonstration of this thing, (if it be required) let us consider, That the first Axiom about Proportion, which is naturally evident, and on which the whole Doctrine

[k] *Ax.* 10. *Fig.* LXII. Doctrine of it is founded, is this, [k] That equal things (A, C,) have the same bigness in respect of a third; (or of equals (B, D). Since in this consideration equals are but *the same*, (for 20 Yards in a bundle, make but one Yard measure.) In this case, (A . B :: C . D) Proportion and Equality is coincident; the Terms being both Equal and Proportional.

Let us now increase the two first of these equal terms, (A and C). And to do it by degrees, (that the way may be clear,) let us first add one point to the measurer B, (as in *b*) (the effect of which is this; that A is not now so big in re-

[l] *Def.* 39. spect of *b*; as C is, in respect of D. $\left(\frac{A}{b} \sqsupset \frac{C}{D}\right)$ and thus the Proportion is destroy'd: to restore which) by consequence we must in *Proportion*, add 3 points to the measured A; (as in *a*)

[m] *Sup.* [m] because A is 3 times as big as B. Thus the Proportion is restored, (and the equality destroyed) *a* being now three times as big as *b*; just as C is to D. (*a* . *b* :: C . D). *Q. E. D.*

To Proceed.

It is naturally evident, in the first 4 terms, that A is as big in respect of C; as B is, in respect of D, (because of their equality) (A . C :: B . D) consider them now increast with the points; which we have already prov'd were added *Proportionably*; (A being 3 times as big as B; and therefore ought to have 3 points for the others one,) so that *a* and *b* are *Proportionably* increast above what they were before; that

that is, *a* is as much bigger than it was (A); as *b* is than it was (B). Or (ſince A, C and B, D are equal) *a* is much bigger than C, as *b* is than D; *a*. C :: *b*. D, which is *Alterned*. *Q. E. D.*

And thus how many points ſoever (or Lines) you add to B, if you add of the ſame to A, in *Proportion* to its bigneſs above B, there will ſtill be the ſame reaſon why *a* ſhould be to C, as *b* is to D. For ſince A and B were *Proportional to C and D; and are now Proportionally increas'd (in *a* and *b*) above what they were before, that is above C, D, it follows that they muſt ſtill be Proportional to C, D; that is, *a*. C :: *b*. D. *Q. E. D.*

*Proportional being eqn.

Theorem LXIII.

Like Parts are to one another, as their Wholes are.

B and D are like parts of the whole ſides of the Triangle, it being prov'd in LXI. 3. that A+B . B :: C+D . D, 21 . 9 :: 14 . 6, therefore *Alternedly*, (LXI. 2.) it ſhall be, A+B . C+D :: B . D, 21 . 14 :: 9 . 6. *Fig. LXI.*

Theorem

Theorem LXIV.

Reasonable Proportion. 1. *In Order.*

If in continued Proportion there be several terms in two rows, A, B, C, ∴ D, E, F, &c. ∴ *Or if several terms be carried on proportionaly, two in one row to two in another*, A . B :: D . E, and B . C :: E . F, &c. *then the extreams* (A, C) *in one row, shall be proportional to the extreams in the other.*

If,

A, B, C, &c. ∴ D, E, F, &c. ∴

Or if,

A . B	::	D . E	
12 . 6	::	18 . 9	
B . C	::	E . F	
6 . 4	::	9 . 6	
&c.		&c.	

Then,

A	. C	:: D	. F
:: 12	. 4	:: 18	. 6

AS may appear from the numbers, becauſe $\frac{C}{4}$ is contain'd in $\frac{A}{12}$ three times, juſt as $\frac{F}{6}$, is in $\frac{D}{18}$. But for further proof, $\frac{A}{C}$ may be call'd

a

a whole; as being compoſed of $\frac{A}{B}$ and $\frac{B}{C}$. The like of $\frac{D}{F}$ as compos'd of $\frac{D}{E}$ and $\frac{E}{F}$. But the parts of both theſe wholes are = (*viz.* $\frac{A}{B}$ ° = $\frac{D}{E}$ ° *Sup.* and $\frac{B}{C}$ ° = $\frac{E}{F}$) and therfore the wholes ($\frac{A}{C}$ & $\frac{D}{F}$) muſt be = too, (*Ax.*5.) *Q. E. D.*

Theorem LXV.

If the former terms remaining, you prefix S *to the ſecond row, ſo that the two laſt* (B, C) *of the firſt row, may be proportional to the two firſt* (S,D) *of the ſecond row, it ſhall be as* A . C :: S . E. 2. *Diſtrib'd*

S.

A, B, C, ∴ D, E, F, ∴

Or if

S.

27

A . B :: D . E
12 . 6 :: 18 . 9

B . C { :: E . F
6 . 4 { :: S.D 9 . 6
27.18

Then,

A . C :: S . E
12 . 4 :: 27 . 9

For

p *Sup.* For, $\frac{E}{F}$ p $= \left(\frac{B}{C}\right.$ p $\left.=\right)\frac{S}{D}$. Therefore by Alter-

q *Pre.* ning and Inverting $\frac{S}{E} = \left(\frac{D}{F}\right.$ q $\left.=\right)\frac{A}{C}$. *Q.E.D.*

It may appear also from the numbers.

Theorem LXVI.

Unproportional terms, are unproportional in all the foregoing Changes.

Fig. LXII. If $\frac{A}{B} \sqsubset \frac{C}{D}$: Then also Alterned, &c. it shall be $\frac{A}{C} \sqsubset \frac{B}{D}$. For suppose the one point only added, to B (as in *b*,) A is now not so big in respect of *b* as C is in respect of D (as we observed) $\left(\frac{A}{b} \sqsupset \frac{C}{D}\right)$ and by consequence *Alternedly* $\frac{A}{C} \sqsupset \frac{b}{D}$; for A is but $=$ C, but *b* is $\sqsubset$ D, by a point. All which is naturaly evident, by looking on the Lines, and may easily be applied to the other Changes. To give one instance in numbers, let it be $\frac{12.A}{9.B} \sqsupset \frac{C.8}{D.5}$ then Alterned, $\frac{12.A}{8.C} \sqsupset \frac{B.9}{D.5}$. For, 8 is contain'd but once and a $\frac{1}{2}$ (*viz.* 4.) in 12, whereas 5 is contain'd almost twice in 9: So that 9 is bigger in respect of 5 than 12 is, in respect of 8, or, what is the same, 12 is $\sqsupset$ in respect of 8, &c.

CHAP.

CHAP. III. PART II.

Of Proportion of Triangles, &c.

Theorem LXVII.

The sides [$\alpha\beta, \alpha\gamma$] *of a Triangle are cut Proportionally by a Line* [$\delta\varepsilon$] *Parallel to the base* [$\beta\gamma$], *viz.* A . B :: C . D.

THE Line $\delta\varepsilon$ being always mov'd Parallel towards $\beta\gamma$, will at last meet with it in all points at once (for if any part of $\delta\varepsilon$ should touch before another, it would have been inclin'd in that part to $\beta\gamma$, and by consequence would not be Parallel) and by consequence in the points β and γ. Since then $\delta\varepsilon$ begins at once upon both the sides of the Triangle $\alpha\beta$, $\alpha\gamma$; and at the same time comes to the end of both, β and γ, it follows, that when it is come to the middle of one, it is also at the middle of the other; when it has past over three parts of one, it has past the like of the other, &c. So that B, D, will be always *like* parts of the sides A + B and C + D, and by consequence A + B . B :: C + D . D, and by dissolving the composition, A . B :: C . D. *Q. E. D.*

Theorem

Theorem LXVIII.

A Line [$\eta\theta$] *cutting the sides of a Triangle Proportionaly, is Parallel to the base* [$\beta\gamma$].

FOr if any part of $\delta\varepsilon$ (that is $\eta\theta$) should touch the base $\beta\gamma$ before another, it would be inclin'd to the base in that part, and by consequence would not be Parallel. *Q. E. D.*

Theorem LXIX.

The base is to the Parallel Line, [$\beta\gamma$] *as the sides are to their parts next the top* [α]: *That is,* E + F . G :: B + A . A :: D + C . C.

FOr $\beta\delta$ being drawn Parallel to C D, it will be (by the Prec. *inverted*) D . C :: B . A
[r] XXX. :: F . (E[r] =) G therefore *compounded* (supposing C D the base, and η, the top) D + C . C :: B + A . A :: ~~F + G . G. Q. E. D.~~
F + E (E.i.e) G

Theorem

Theorem LXX.

A Parall. to the base, cuts off a part [X], *towards the top*, Like [::] *to the whole Triangle.* *Def.* 45.

FOr the Angles $\left\{\begin{matrix}\alpha\gamma\beta = \alpha\theta n\\ \alpha\beta\gamma = \alpha n\theta\end{matrix}\right\}$ and α is common, therefore X is equiangled to the whole Triangle. [r] *IV.*

Further, the side [s] DC. C :: FE. G :: BA. A: Therefore *Alterned*, DC. FE :: C. G
and FE. BA :: G. A:
Lastly because B. A :: [t] D. C, therefore *Compounded*, B + A. A :: D + C. C, and *Alterned* B + A. D + C :: A. C: Thus all the sides about the equal Angles are proportional; and by [u] consequence X is :: to the whole Triangle. *Q. E. D.* [s] *Pre.* [t] *LXVII. Inverted.* [u] *Def.* 45.

Theorem LXXI.

Equiangled Triangles [$\alpha n\theta$, S.] *are* ::

CUt off, by the line $\beta\gamma$; $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$, because the Angle α [x] $= \delta$, therefore X $=$ S, therefore the Angle γ [y] $= (\zeta$ [x] $=)\theta$, therefore $\beta\gamma$ [z] Parall. $n\theta$; and by consequence the Triangle $\alpha n\theta$, is [a] :: (X [b] ::) S. *Q. E. D.* [x] *Sup.* [y] *XVII.* [z] *VI.* [a] *Pre.* [b] *Ax.* 10.

Theorem LXXII.

Triangles [$\alpha n \theta$, S.] *are like, whose ſides are Proportional.*

TAke $\alpha\gamma = \delta\zeta$, and draw $\gamma\beta$ Parall. θn: Therefore $\frac{B+A}{A}$ [c] $= \left(\frac{D+C}{C \text{ [d] or } \delta\zeta}\right)$ [e] $= \frac{B+A}{\delta\varepsilon}$: therefore, A [f] $= \delta\varepsilon$: Further, $\frac{E}{F}$ [g] $= \left(\frac{D+C}{C \text{ [h] or } \delta\zeta}\right)$ [e] $= \frac{E}{\varepsilon\zeta}$: Therefore F [f] $= \varepsilon\zeta$; ſo then S has equal ſides to X, and by [h] conſequence = Angs. therefore is [i] :: (X [k] ::) $\alpha n \theta$. *Q. E. D.*

[c] *LXVII.*
[d] *Conſtr.*
[e] *Hyp. Alterned.*
[f] *Ax.* 10.
[g] *LXIX.*
[h] *XVI.*
[i] *Pre.*
[k] *LXX.*

Theorem LXXIII.

Triangles [$\alpha n \theta$, S.] *whoſe ſides are Proportional, are equiangled.*

THis is Demonſtrated in the operation of the Precedent; for the Suppoſition being the ſame [l] [illegible] Angles to (X, which has [l] = Angles to) $\alpha n \theta$. *Q. E. D.*

[l] *LXX. and*

Theorem LXXIV.

Triangles [$\alpha\eta\theta$, S.] *are* ::, *which have one Angle* =, [$\alpha = \delta$] *and the sides about that Angle Proportional* B+A . C+D :: $\delta\varepsilon . \delta\zeta$.

CUt off, (by the Line $\beta\gamma$) $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$. Because the Angle $\alpha^m = \delta$, therefore $X^n = S$, and by [o] consequence is ::, now because A+B. ($\delta\varepsilon^p$ or) A^m :: C+D. ($\delta\zeta^p$ or) C: Therefore [q] $\beta\gamma$ Parallel $\eta\theta$, and by consequence $\alpha\eta\theta$ is [r] :: (X ::) S. *Q. E. D.*

m *Sup.*
n *XVII.*
o *LXXI.*
q *Const.*
q *LXVIII.*
r *LXVII.*

Theorem LXXV.

Triangles [$\alpha\eta\theta$, S.] *are* ::, *which have two sides Proportional* [B+A . D+C :: $\delta\varepsilon . \delta\zeta$], *and the Angle opposite to these sides,* = [$\theta = \zeta$] *and another of the same kind.*

CUt off (by the Line $\beta\gamma$) $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$. Because $\frac{B+A}{(\delta\zeta^s \text{ or})A}{}^s = \frac{D+C}{(\delta\varepsilon^t \text{ or})C}$: [u] There-

s *Sup. Alternad.*
t *Constr.*
u *LXVIII.*

For, $\frac{E}{F}$ [p] $=$ $\left(\frac{B}{C}\right.$ [p] $=\left.\right)\frac{S}{D}$. Therefore by Alternating and Inverting $\frac{S}{E}=\left(\frac{D}{F}\right.$ [q] $=\left.\right)\frac{A}{C}$. *Q.E.D.*

p *Sup.*

q *Pre.*

It may appear also from the numbers.

Theorem LXVI.

Unproportional terms, are unproportional in all the foregoing Changes.

Fig. LXII. If $\frac{A}{B}\sqsubset\sqsupset\frac{C}{D}$: Then also Alterned, *&c.* it shall be $\frac{A}{C}\sqsubset\sqsupset\frac{B}{D}$. For suppose the one point only added, to B (as in *b*,) A is now not so big in respect of *b* as C is in respect of D (as we observed) $\left(\frac{A}{b}\sqsupset\sqsubset\frac{C}{D}\right)$ and by consequence *Alternedly* $\frac{A}{C}\sqsupset\sqsubset\frac{b}{D}$; for A is but $=$ C, but *b* is $\sqsubset$ D, by a point. All which is naturaly evident, by looking on the Lines, and may easily be applied to the other Changes. To give one instance in numbers, let it be $\frac{12.A}{9.B}\sqsupset\sqsubset\frac{C.8}{D.5}$ then Alterned, $\frac{12.A}{8.C}\sqsupset\sqsubset\frac{B.9}{D.5}$. For, 8 is contain'd but once and a $\frac{1}{2}$ *(viz.*4.) in 12, whereas 5 is contain'd almost twice in 9: So that 9 is bigger in respect of 5 than 12 is, in respect of 8, or, what is the same, 12 is $\sqsupset$ in respect of 8, *&c.*

CHAP.

CHAP. III. PART II.

Of Proportion of Triangles, *&c.*

Theorem LXVII.

The ſides [$\alpha\beta, \alpha\gamma$] *of a Triangle are cut Proportionally by a Line* [$\eta\theta$] *Parallel to the baſe* [$\beta\gamma$], *viz.* A. B :: C. D.

THe Line $\delta\varepsilon$ being always mov'd Parallel towards $\beta\gamma$, will at laſt meet with it in all points at once (for if any part of $\delta\varepsilon$ ſhould touch before another, it would have been inclin'd in that part to $\beta\gamma$, and by conſequence would not be Parallel) and by conſequence in the points β and γ. Since then $\delta\varepsilon$ begins at once upon both the ſides of the Triangle $\alpha\beta$, $\alpha\gamma$; and at the ſame time comes to the end of both, β and γ, it follows, that when it is come to the middle of one, it is alſo at the middle of the other; when it has paſt over three parts of one, it has paſt the like of the other, *&c.* So that B, D, will be always *like* parts of the ſides A + B and C + D, and by conſequence A + B . B :: C + D . D, and by diſſolving the compoſition, A . B :: C . D. *Q. E. D.*

Theorem

Theorem LXVIII.

A Line [ν δ] *cutting the ſides of a Triangle Proportionaly, is Parallel to the baſe* [βγ].

FOr if any part of δ ε (that is ν δ) ſhould touch the baſe β γ before another, it would be inclin'd to the baſe in that part, and by conſequence would not be Parallel. *Q. E. D.*

Theorem LXIX.

The baſe is to the Parallel Line, [βγ] *as the ſides are to their parts next the top* [α]: *That is,* E + F . G :: B + A . A :: D + C . C.

FOr β δ being drawn Parallel to C D, it will be (by the Prec. *inverted*) D . C :: B . A
= XXX. :: F . (E =) G therefore *compounded* (ſuppoſing C D the baſe, and ν, the top) D + C . C :: B + A . A :: ~~E + G . G~~ ~~*Q. E. D.*~~
F + E (E. i. e) G

Theorem

Theorem LXX.

A Parall. to the baſe, cuts off a part [X], *towards the top*, Like [::] *to the whole Triangle.* Def. 45.

FOr the Angles $\left\{\begin{matrix}\alpha\gamma\beta\\ \alpha\beta\gamma\end{matrix}\right\}^{r} = \left\{\begin{matrix}\alpha\theta\eta\\ \alpha\eta\theta\end{matrix}\right\}$ and α is common, therefore X is equiangled to the whole Triangle. [r] IV.

Further, the ſide[r] DC. C :: FE. G :: BA. A: Therefore *Alterned*, DC. FE :: C. G and FE. BA :: G. A: [r] Pre.

Laſtly becauſe B. A :: [u] D. C, therefore *Compounded*, B + A. A :: D + C. C, and *Alterned Inverted*. B + A. D + C :: A. C: Thus all the ſides about the equal Angles are proportional; and by [w] conſequence X is :: to the whole Triangle. *Q. E. D.* [u] LXVII. [w] Def. 45.

Theorem LXXI.

Equiangled Triangles [$\alpha\eta\theta$, S.] *are* ::

CUt off, by the line $\beta\gamma$; $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$, becauſe the Angle α [x] = δ, therefore X = S, therefore the Angle γ [y] = (ζ [x] =) θ, therefore $\beta\gamma$ [z] Parall. $\eta\theta$; and by conſequence the Triangle $\alpha\eta\theta$, is[a] :: (X [b] ::) S. *Q. E. D.* [x] Sup. [y] XVII. [z] VI. [a] Pre. [b] Ax. 10.

Theorem LXXII.

Triangles [$\alpha n \theta$, S.] *are like, whose ſides are Proportional.*

TAke $\alpha\gamma = \delta\zeta$, and draw $\gamma\beta$ Parall. θn: Therefore $\frac{B+A}{A}$ [c] $= \left(\frac{D+C}{C \text{ [d] or } \delta\zeta}\right)$ [e] $= \frac{B+A}{\delta\varepsilon}$: therefore, A [f] $= \delta\varepsilon$: Further, $\frac{E}{F}$ [g] $= \left(\frac{D+C}{C \text{ [h] or } \delta\zeta}\right)$ [e] $= \frac{E}{\varepsilon\zeta}$: Therefore F [f] $= \varepsilon\zeta$; ſo then S has equal ſides to X, and by [h] conſequence = Angs. therefore is [i] :: (X [k] ::) $\alpha n \theta$. *Q. E. D.*

[c] *LXVII.* [d] *Conſtr.* [e] *Hyp. Alterned.* [f] *Ax.* 10. [g] *LXIX.* [h] *XVI.* [i] *Pre.* [k] *LXX.*

Theorem LXXIII.

Triangles [$\alpha n \theta$, S.] *whoſe ſides are Proportional, are equiangled.*

THis is Demonſtrated in the operation of the Precedent; for the Suppoſition being the ſame in both, it is there prov'd, that S has = Angles to (X, which has [l] = Angles to) $\alpha n \theta$. *Q. E. D.*

[l] *LXX. and Def.* 45.

Theorem

Theorem LXXIV.

Triangles [$\alpha \eta \theta$, S.] *are* ::, *which have one Angle* =, [$\alpha = \delta$] *and the ſides about that Angle Proportional* B+A. C+D :: $\delta\varepsilon . \delta\zeta$.

CUt off, (by the Line $\beta\gamma$) $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$. Becauſe the Angle $\alpha^{m} = \delta$, therefore $X^{n} = S$, and by ° conſequence is ::, now becauſe A+B. ($\delta\varepsilon$ P or) A^{m}:: C+D. ($\delta\zeta$ P or) C: Therefore $\beta\gamma$ Parallel $\eta\theta$, and by conſequence $\alpha\eta\theta$ is ^{r}:: (X::) S. *Q. E. D.*

m *Sup.* n *XVII.* o *LXXI.* q *Conſt.* q *LXVIII.* r *LXVII.*

Theorem LXXV.

Triangles [$\alpha \eta \theta$, S.] *are* ::, *which have two ſides Proportional* [B+A. D+C :: $\delta\varepsilon . \delta\zeta$], *and the Angle oppoſite to theſe ſides,* = [$\theta = \zeta$] *and another of the ſame kind.*

CUt off (by the Line $\beta\gamma$) $\alpha\beta, \alpha\gamma = \delta\varepsilon, \delta\zeta$. Becauſe $\frac{B+A}{(\delta\zeta^{s}\text{or})A}{}^{t} = \frac{D+C}{(\delta\varepsilon^{t}\text{or})C}$: ^{u} Therefore $\beta\gamma$ Parallel $\eta\theta$. Now the Angle $\zeta^{s} = (\theta^{w} =)\gamma$: Therefore $X^{x} = S$, and by conſequence ^{y}:: , but $\alpha\eta\theta$ ^{z}:: (X::) S. *Q. E. D.*

s *Sup. Alterned.* t *Conſtr.* u *LXVIII.* w *VI.* x *XXIV.* y *LXXI.* z *LXVII.*

Theorem LXXVI.

A Right-angled Triangle [a b c], *divided by a Perpendicular* [a d] *from the* ∟ *to the ſubtendent, is* :: *its parts* [a b d, a d c].

FOr each part has one Angle common to the whole, *viz.* B and C, and one Angle ∟ (*viz.* at D,) [a] = B A C: Therefore the whole is [b] equiangled to its parts, and by conſequence [c] is :: to them. *Q. E. D.*

[a] *Ax.* 2. [b] *XXII.* [c] *LXXI.*

Thorem LXXVII.

The ſides of a Triangle are Proportional to the parts of its baſe, divided by a Line that cuts the oppoſite Angle in the middle f g. A :: D. C.

DRaw out A, Infinitly, and G I Parallel F H, therefore the Angle F G I [d] = (H F G [e] = H F E [d] =) G I F, and therefore the ſubtendent F I [f] = F G. [g] But, (B that is) F G. A :: D. C. *Q. E. D.*

[d] *IV.* [e] *Sup.* [f] *XII.* [g] *LXVII.*

Theorem

Theorem LXXVIII.

Triangles of the ſame height, (or between the ſame Parallels) are to one another as their baſes, that is, S . Z :: DC . CF.

TAke DB = CF, therefore X[h] = Z. Now if CA be mov'd, on the point A, as on a Center, towards AB, they will at laſt meet: and CA ſhall at the ſame time have paſt over the whole Triangle, (ABC,) and the baſe (BC;) and by conſequence when it has come to the middle of one, it will be at the middle of the other; and ſo, in Proportion, whatſoever part it has paſt over of the Triangle, it will have paſt over a like of the baſe; and (by conſeq.) will divide the Triangle and baſe Proportionally; that is, that S ſhall be to (X =) Z :: DC. (DB =) CF. *Q. E. D.* [h] *XXVI.*

Theorem LXXIX.

Equal Triangles X, S, *having one* = *Ang.* [*at* C] *have their ſides about this Ang. reciprocally proportional,* AC . CE :: DC . CB.

LEt the = Angles ECD, BCA, be ſo joyned that EA, BD may be Right-lines, (which is poſſible;

possible; as may easily appear from III. as the II. does from I. Then join BE: Now,

i *Pre.* k *For* X = S *Sup.*

$$\frac{AC}{CE} \overset{i}{=} \left(\frac{X \overset{k}{=} S}{Z \text{ (Ax. 10.) } Z} \overset{i}{=}\right) \frac{DC}{CB}. \quad Q. E. D.$$

Theorem LXXX.

Parallelograms [O, S,] *of the same height, are to one another, as their bases* [c d, d g].

l *XXX.* m *LXXVIII* n *Ax. 7.*

For the l Triangle CD, is $\frac{1}{2}$ O; also DFG, is $\frac{1}{2}$ S. But $\frac{CD}{\frac{1}{2}O} \overset{m}{=} \frac{DG}{\frac{1}{2}S}$, and *Alternè*

$$\frac{CD}{DG} = \left(\frac{\frac{1}{2}O}{\frac{1}{2}S} \overset{n}{=} \frac{\text{whole}}{\text{whole}}\right) \frac{O}{S}. \quad Q. E. D.$$

Theorem LXXXI.

Parallelograms [O, S,] *having one Angle* =, [*at*, c] *have their sides about this Angle reciprocally proportional*, EC. CA :: BC. CD.

o *Sup.* p *LXXII.* q *LXXIX.*

For since S o = O, therefore $\frac{1}{2}$ S (*viz.* the Triangle ECD) p = $\frac{1}{2}$ O, (*viz.* BCA:) and q therefore, EC. CA :: BC. CD. *Q. E. D.*

The orem

Theorem LXXXII.

Parallelograms [S, O,] *that have one Ang.* =, *(and by conſequence all Rectangles) are to one another in a reaſon* [r] *compounded of that of their ſides.* [r] *Def.* 43.

FOr ſince ſuch Figures have both Length and Breadth, to know the *reaſon* of one to the other, it is neceſſary to compare both their Length and Breadth, (that is, their ſides) and out of the *reaſon* of both theſe, is *compounded* the *reaſon* of the Figures. For inſtance, if S be twice as long, and thrice as broad as O, then S is two times thrice, (that is, 6 times) as big as O, (as may be ſeen by the prickt Lines, in S,) ſo that $\frac{S}{O} = \frac{DB}{BA} + \frac{EB}{BC}$. *Q. E. D.*

Theorem LXXXIII.

Like *Parallelograms* [S, O,] *are in a Duplicate* reaſon *to that of their Homologous ſides*, ĐB, BA, EB, BC.

THe reaſon of Pgrs. as we have ſaid, is *(compounded* or,) made up of the *reaſons* of their Length and Breadth. But now in *Like* Pgrs. the *reaſons* of their Length and Breadth,

are the ſame, (that is, S is juſt as much broader, as it is longer, than O:) So that, whereas in the former Propoſition, to find the reaſon of S to O, we were to joyn together the reaſons of their Length and Breadth; here it will be ſufficient to take the *reaſon* of either, twice; becauſe they are both the ſame: Thus, if S be twice as long, and (by conſequence) twice as broad as O, then it is two times twice (that is 4 times) as big as O, and this is call'd *Duplicate* reaſon. For Lines thus, $\frac{DB}{EB}$ [f] $= \frac{BA}{BC}$, and *Alternedly*, $\frac{DB}{BA} = \frac{EB}{BC}$: But $\frac{S}{O}$ [t] $= \left(\frac{DB}{BA} + \frac{EB}{BC} =\right)$ $2\frac{DB}{BA}$ or, $2\frac{EB}{BC}$. *Q. E. D.* For further Illuſtration,

[f] *Sup.*

[t] *Pre.*

Fig. II. If the Length of S, be to the Length of O, as 4 to 3, by [u] conſequence the Breadth muſt be as 4 to 3 alſo. Then $\frac{S}{O}$ [w] $= \frac{4}{3} + \frac{4}{3}$, that is, $= 2\frac{4}{3}$: *viz.* $\frac{16}{9}$, as appears by the little Squares in S and O.

[u] *Def.* 42.

[w] *Pre.*

Theorem LXXXIV.

Parallelograms [S, O,] *are equal, which have one Angle* =, [α, β,] *and the ſides about this, reciprocally proportional.*

Viz. If A . D :: C . B, then S = O. Now *Alterned*, A . C :: D . B; that is, S is as much

much longer than O; as O is broader than S, and [x] ſince the bigneſs of S to O, is made up of the bigneſs of the ſides, if S be 3 times longer than O; and O 3 times broader than S, theſe two reaſons in being *compounded* deſtroy one another, and S is left = O. *Q. E. D.* [x] *LXXXII.*

Theorem LXXXV.

All Squares are :: *(Figures, or) Pgrs.*

FOr A [y] = B and C [y] = D, therefore, A . C :: [z] B . D, and *Alterned*; A . B :: C . D: Therefore S is [a] :: O. *Q. E. D.*

[y] *Sup.* [z] *Ax.* 10. [a] *Def.* 45.

Theorem LXXXVI.

Like Rect-angles [S, O,] *are to one another as the Squares of their Homologous ſides*, A . B.

FOr $\frac{S}{O}$ [b] = (2 $\frac{A}{B}$ [c] =) $\frac{Aq}{Bq}$. *Q. E. D.*

[b] *LXXXIII* [c] *Pre. with LXXXIII.*

Theorem

Theorem LXXXVII.

Triangles that have one Angle = , *are to one another in a reaſon compounded of that of their ſides.*

F.LXXXII

FOr Triangles are [d] ½ Pgrs. (Diameters being drawn from E to D, and from A to C,) and likewiſe receive the ſame ſides with S and O.

But $\frac{\frac{1}{2}S}{\frac{1}{2}O}$ [e] $= \left(\frac{S}{O}\right.$ [f] $= \left.\right) \frac{DB}{BA} + \frac{EB}{BC}$. *Q. E. D.*

d XXX.

e LXIII.

f LXXXII.

Theorem LXXXVIII.

Like Triangles, are in a Duplicate reaſon to that of their Homologous ſides.

f LXXXIII

FOr theſe, again, are ½ the Pgrs. S and O:

Therefore $\frac{\frac{1}{2}S}{\frac{1}{2}O}$ [g] $= \left(\frac{S}{O}\right.$ [h] $= \left.\right) 2\frac{DB}{BA}$ or $2\frac{EB}{BC}$. *Q. E. D.*

g LXIII.

h LXXXII.

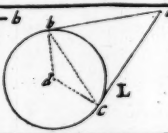
Theorem

XLIII

XLIV

XLV

XLIX



L

LI

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LVI

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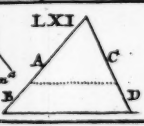
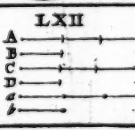
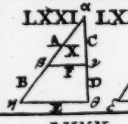


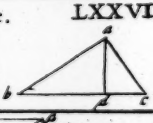
LXII



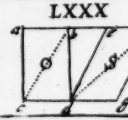
LXXI. LXXII. &c.



LXXVI



LXXX



LXXXI

V

XLVI

XLVII

XLVIII

LI

LII

LIII

LIII

LVI

LVII

LVIII

LIX

II

LXVII

LXVIII

LXIX

LXX

LXXVII

LXXVIII

LXXIX

LXXXII

LXXXIII 2.

LXXXIII.

Face to p. 70.

Theorem LXXXIX.

Like *Pgrs.* [a d, e g] *plac'd* ::, *and having one common Angle* [c], *receive the ſame Diameter* [c b].

IF not, let the inner Pgr. be CFHG; and draw F G parallel D B: Therefore, the Angle C G F[i] = (D[k] =) C G H, (for C E H is ſuppoſed :: to A D.) The like, if H fall on the other ſide the Diameter, therefore ſince H can fall on neither ſide the Diameter, it muſt fall upon it. *Q. E. D.*

[i] *IV.*
[k] *Def.*

Theorem XC.

Like *Figures* [a b c d e, f g h i k,] *of many ſides, may be divided into an equal number, of* :: *Triangles, by the Lines* [a c, a d, f h, f i].

THe firſt part is plain only by looking on the Figures; the ſecond, that the Triangles are ::, is prov'd thus: The Angle B = G; and the ſides, A B. B C :: [l] F G. G H: Therefore the Triangle A B C[m] like F G H; and by the ſame reaſon A E D like F K I. Laſtly C A D[n] like H F I; (for the whole Figures were [l] equiangled, and equal Angles have been taken away on [o] each ſide, therefore there remains = Angs. to C A D and H F I. *Q. E. D.* Theorem

[l] *Sup.*
[m] *LXXIV.*
[n] *LXXI.*
[o] *Being prov'd* ::

Theorem XCI.

Like *Figures* [a b c d e, f g h i k,] *are to one another, in a* Duplicate *reason (or as the Squares) of their Homologous sides.*

[p] *XXX.* [q] *LXXXIII.* [r] *LXXXV.* [s] *Pre.* FOr (every Triangle being [p] ½ a Pgr.) the [r] :: Triangles $\frac{ABC}{FGH}$ [q] $= 2\frac{AB}{FG}$ ([r] $= \frac{ABq}{FGq}$) (and $\frac{AED}{FKI} = 2\frac{AE}{FK}$ and $\frac{CAD}{HFI} = 2\frac{CD}{HI}$. But now, because AB.BC :: FG.GH, BC.CD :: GH.HI } therefore [t] AB . CD :: FG . HI; and by the same reason AE . CD :: FK . HI; and *Alternè* of both together $\frac{AB}{FG} = \frac{CD}{FI} = \frac{AE}{FK}$: Since then the reason of all these Homologous sides, is equal, the Triangles $\frac{ABC}{FGH}$ or $\frac{AED}{FKI}$ or $\frac{CAD}{HFG}$ [u] $= 2\frac{AB}{FG}$)

[t] *LXIV.* [u] *Ax.* 10. [w] *LXIII.* Therefore the wholes $\frac{ABCDE}{FGHIK}$ [w] $= 2\frac{AB}{FG} = \frac{ABq}{FGq}$. *Q. E. D.*

Theorem

Theorem XCII.

Circles are to one another in a Duplicate *reaſon of their Diameters, (or, as the Squares of Diameters* LXXXVI. *with* LXXXIII.)

THat is $\frac{\bigcirc, ADF}{\bigcirc, \alpha \delta \varphi} = 2\frac{AB}{\alpha\beta} = \frac{AB\,q}{\alpha\beta\,q}$. If not, let $\frac{\bigcirc, ADF}{\bigcirc, \alpha \delta \varphi} \sqsubset 2\frac{AB}{\alpha\beta}$. And in the $\bigcirc$, ADF, let there be in inſcrib'd a Polygon (or many ſided Figure) Z in ſuch manner that $\frac{Z}{\bigcirc, \alpha \delta \varphi} \sqsubset 2\frac{AB}{\alpha\beta}$ (which appears to be poſſible, becauſe the ſides of the Polygon may be ſo multiplied 'till they come to be = Circumference of the $\bigcirc$ it ſelf, and by conſequence whatſoever the $\bigcirc$ exceeds, Z may likewiſe exceed.) Then in the $\bigcirc, \alpha\beta\varphi$, let the Polygon X be * inſcrib'd, :: Z. Now the $\triangle$, AFB is [x] :: $\alpha\varphi\beta$; and by conſequence $\frac{AFB}{\alpha\varphi\beta} = [y]\ 2\frac{AB}{\alpha\beta}$ And. $\frac{Z}{X}$ [z] $= \left(\frac{AFB}{\alpha\varphi\beta}\right.$ $\left. = \right) 2\frac{AB}{\alpha\beta}$: But $\frac{Z}{\bigcirc, \alpha\delta\beta}$ [a] $\sqsubset 2\frac{AB}{\alpha\beta}$. [b] Therefore X $\sqsubset \bigcirc, \alpha\delta\beta$. [c] *Q. E. A.* Therefore $\frac{\bigcirc, ADF}{\bigcirc, \alpha \delta \varphi} = 2\frac{AB}{\alpha\beta}$ or $\frac{AB\,q}{\alpha\beta\,q}$. *Q. E. D.*

* *See Problem.* [x] *XC.* [y] *Pre.* [z] *LXXIII.* [a] *Conſtr.* [b] *Ax.* 11. [c] *Ax.* 5.

Theorem

Theorem XCIII.

All the external Angles of any Polygon are equal 4 ∟ Angles.

1. A Polygon may be divided into so many Triangles, as it has sides, by Lines drawn from any point within, to the several Angles; this appears by looking on the Figure.

2. And by consequence it contains twice as many [d] ∟ Angles, (as it has sides;) except [e] 4, which are about the point in the middle, (for all the Angles about a point are = 4 ∟).

[d] *X.*
[e] *I.*

3. Then, drawing out the several sides; Each against the other makes 2 Angles [d] = 2 ∟; and by consequence all the Angles together, inward and outward, will be = to twice as many ∟, as there are sides.

[f] *XCIII*, 2.

4. But the inward Angles are prov'd = to these, except 4, [f] and therefore the outward Angles remain = 4 ∟. *Q. E. D.*

Theorem XCIV.

Three only Regular *Figures (that is, whose sides and Angles are =) can fill a Space, viz. Triangles, as in* A, *Squares, as in* C, *and Hexagons, as in* B.

FOr since all the Angles about a point are [g] = 4 ∟.

[g] *I.*

1. The

1. The Angle of a *regular* △ is [h] = ⅔ 2 ∟ (becauſe all the 3 Angles are =) and by conſequence 6 of theſe joyn'd together, as in A, are = 4 ∟ Angles, for ⅔ 2 ∟ = 4 ∟. [h] *X.*

2. The Angle of the □ is [i] ∟; and by conſequence the 4 at C are = 4 ∟. [i] *Def.*

3. The Angle of a Hexagon is [k] = 1 and ⅓ ∟, and by conſequence the 3 joyn'd together in B are = 3 and ⅓ ∟, that is, 4 ∟. [k] *Pro.* 2.

Now all Figures that have more ſides than this, have their Angles bigger than it, and by conſequence 3 of them (which is the [g] leaſt *(number)* that can make a plane Angle) will be ⊏ 4 ∟; In the Pentagon (of 5 ſides) 3 Angles will be ⊐, 4 ⊏ than [k] 4 ∟. *Q. E. D.*

Theorem XCV.

Like *Figures,* [S, O, Z, X,] *fram'd a like upon Proportional baſes,* [a. b :: c. d] *will be themſelves alſo Proportional,* viz. S. O :: Z. X.

FOr, $\frac{S}{O}$ [l] $= \left(\frac{A}{B} \overset{m}{=} \frac{C}{D}\right) = \frac{Z}{X}$. *Q. E. D.* [l] *XCI.* [m] *LXIII, for A. B :: C. D, Sup.*

Theorem XCVI.

In a Circle, [b a c, b a d,] *the Angles are Proportional to the Arches* [b c, b d,] *that ſubtend them.*

FOr the Line AB, being mov'd towards C, at the ſame time will diſpatch both the Angle

A, and the Arch BC; and by consequence when it comes to the middle of one, it will be at the middle of the other, and so forward; so that in fine, whatsoever part BAD will be of BAC, BD will be the same part of BC: Therefore the Angle BAD, will be to BAC :: Arch B, to BC. *Q. E. D.*

Theorem XCVII.

A Polygon conscrib'd about a Circle is = to a Right Angled Triangle, whose base [$\beta\beta$] *is the Compass of the Polygon, and its height* [$\alpha\beta$] *the Radius of the Circle.*

DRaw the Radius's AH, AI, &c. to the sides of the Polygon at the point of touching, then take $\beta H = BH$; and $HC = HC$; and so forward, 'till you come to NB; then draw αZ parallel $\beta\beta$. Erect the Perpendiculars, HO, IP, KQ, &c. and joyn Oβ, OC; PC, PD, &c. Now, because the Angle OHB [n] = ($\llcorner$ [o] =) $\alpha\beta H$, therefore HO is [p] parallel to $\alpha\beta$; and by [q] consequence HO = ($\alpha\beta$ [o] =) HA; therefore the $\triangle$s, BAC and βOC have the same height; therefore BAC [r] = (βOC [s] =) βAC: In like manner CαD = (CPD =) CAD. &c. and, in fine, G$\alpha\beta$ = (GZB =) GAB. So then all the parts being equal, the whole $\triangle \alpha\beta\beta$ [t] = to the Polygon. *Q. E. D.*

[n] *Constr.*
[o] *Sup.*
[p] *IV.*
[q] *XXX.*
[r] *XXVIII.*
[s] *XXVI.*
[t] *Ax.* 3.

Theorem

Theorem XCVIII.

Everry regular *Polygon is equal to a Right Angl'd* △, *whoſe baſe is the Compaſs of the Polygon, and its height, a Perpendicular* [a h] *to the ſide of the Polygon* [b c] *from the Center.*

FOr that Perpendicular *(a b)* will be the ſame with the Radius of the Circle that may be inſcribed in ſuch a Polygon, and than the reſt follows from the Precedent.

Theorem XCIX.

A Circle is = *to a* ⌊ *Angled* △, *whoſe baſe is the Circumference of the Circle, and its height, the Radius of it.*

1. EVery [u] Polygon conſcrib'd is ⊏, and inſcrib'd is ⊐, then the ◯. [u] *Ax.* 5.

2. The compaſs of a Polygon conſcrib'd, is ⊏, and inſcrib'd is ⊐, than the [u] circumference of the Circle.

3. This ⌊ Angled △ will be [w] ⊐ than any Polygon conſcrib'd, and ⊏ than any inſcrib'd, (becauſe the circumference of the ◯, which is the baſe of this Triangle is [x] ⊏ than the compaſs of any inſcrib'd.) Therefore it will be = to the ◯. [w] *XCVII.* [x] *Ax.* 5.

For if this Triangle be ⊏ than any thing that is bigger than the ○, and ⊐ than any thing that is ⊐ than the ○, it follows that it muſt be equal to the ○. *Q. E. D.*

Note. And this is call'd the Squaring of a Circle, *viz.* to find a Right-lined figure equal to a Circle; upon this ſuppoſition, that the baſis given is equal to the circumference of the Circle. But actually to find a Right-line = to the circumf. of a ○, is not yet diſcover'd Geometrically.

Theorem C.

An equal Legg'd Triangle [abc] *in a Segment, is* ⊏ *than* ½ *the Segment.*

DRaw the Radius FB Perpend. AC, and the Tangent DBE, which will be Parallel to AC, (becauſe the Angle AGB[y] is ∟ [z] = DBG) further draw AD and CE Parallel GB; DC will be a Pgr. and the Triangle ABC half of it; but the whole Pgr. is ⊏ than the Segment ABC; therefore ½ Pgr. *viz.* ABC, ⊏ [a] ½ the Segment. *Q. E. D.*

[y] *Conſt.*
[z] *XLIX.*
[a] *LXIII.*

Theorem

Theorem CI.

The Triangle DCB *is* ⊏ *than* ½ *the mixt Triangle* ACB.

LEt AB and DC be Tangents: Therefore DC = [b] DA, and Angle (DCF, [c] is ∠ [d] = DCB; therefore the ſubtend. DB [e] ⊏ (DC =) DA: Therefore the Triangle DCB [f] ⊏ (DCA [g] ⊏) ADC, the Triangle mixt, and by conſequence is more than ½ ABC the Triangle mixt. *Q. E. D.*

[b] *L.*
[c] *XLIX.*
[d] *I.*
[e] *XV.*
[f] *LXXVIII*
[g] *Ax.* 5.

Note. From the two Preceding Propoſitions it appears, that figures inſcrib'd and conſcrib'd, if the number of their ſides be doubled, do approach the Circle by more than half the ſpace that was before left.

Theorem CII.

A Circle contains more ſpace, than any other figure that is equal to a Circle.

FOr a ○ is = [h] ∟ Angled △ whoſe baſe is the circumference of the ○, and height, the Radius AB. But the Square CD (which we ſuppoſe = to the ○) is [i] = to ∟ Angl'd △, whoſe baſe indeed, is the ſame with the former, (becauſe the compaſs of the Square is ſuppoſed = circumference of the ○:) but the height is the Perpendicular AE. *Q. E. D.*

[h] *XCIX.*
[i] *XCVIII.*

A, and the Arch BC; and by consequence when it comes to the middle of one, it will be at the middle of the other, and so forward; so that in fine, whatsoever part BAD will be of BAC, BD will be the same part of BC: Therefore the Angle BAD, will be to BAC :: Arch B, to BC. *Q. E. D.*

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A Polygon conscrib'd about a Circle is = to a Right Angled Triangle, whose base [$\beta\beta$] *is the Compass of the Polygon, and its height* [$\alpha\beta$] *the Radius of the Circle.*

DRaw the Radius's AH, AI, &c. to the sides of the Polygon at the point of touching, then take $\beta H = BH$; and $HC = HC$; and so forward, 'till you come to NB; then draw αZ parallel $\beta\beta$. Erect the Perpendiculars, HO, IP, KQ, &c. and joyn OB, OC; PC, PD, &c. Now, because the Angle OHB[n] = ($\angle$ [o] =) $\alpha\beta H$, therefore HO is [p] parallel to $\alpha\beta$; and by [q] consequence HO = ($\alpha\beta$ [o] =) HA; therefore the $\triangle$s, BAC and βOC have the same height; therefore BAC[r] = (βOC [f] =) βAC: In like manner CαD = (CPD =) CAD, &c. and, in fine, G$\alpha\beta$ = (GZB =) GAB. So then all the parts being equal, the whole $\triangle \alpha\beta\beta$ [r] = to the Polygon. *Q. E. D.*

[n] *Constr.*
[o] *Sup.*
[p] *IV.*
[q] *XXX.*
[r] *XXVIII.*
[f] *XXVI.*
[r] *Ax.* 3.

Theorem

Theorem XCVIII.

Every regular *Polygon is equal to a Right Angl'd* △, *whoſe baſe is the Compaſs of the Polygon, and its height, a Perpendicular* [a h] *to the ſide of the Polygon* [b c] *from the Center.*

FOr that Perpendicular *(a b)* will be the ſame with the Radius of the Circle that may be inſcribed in ſuch a Polygon, and than the reſt follows from the Precedent.

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3. This ∟ Angled △ will be [w] ⊐ than any Polygon conſcrib'd, and ⊏ than any inſcrib'd, (becauſe the circumference of the ○, which is the baſe of this Triangle is [x] ⊏ than the compaſs of any inſcrib'd.) Therefore it will be = to the ○. [w] *XCVII.* [x] *Ax.* 5.

For if this Triangle be ⊏ than any thing that is bigger than the ○, and ⊐ than any thing that is ⊐ than the ○, it follows that it must be equal to the ○. *Q. E. D.*

Note. And this is call'd the Squaring of a Circle, *viz.* to find a Right-lined figure equal to a Circle; upon this suppoſition, that the baſis given is equal to the circumference of the Circle. But actually to find a Right-line = to the circumf. of a ○, is not yet diſcover'd Geometrically.

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[y] *Conſt.*
[z] *XLIX.*
[a] *LXIII.*

Theore

Theorem CI.

The Triangle DCB *is* ⌐ *than* $\frac{1}{2}$ *the mixt Triangle* A C B.

LEt A B and D C be Tangents: Therefore D C = [b] D A, and Angle (D C F, [c] is ∟ [d] = D C B; therefore the ſubtend. D B [e] ⌐ (D C =) D A: Therefore the Triangle D C B [f] ⌐ (D C A [g] ⌐) A D C, the Triangle mixt, and by conſequence is more than $\frac{1}{2}$ A B C the Triangle mixt. *Q. E. D.*

[b] *L.* [c] *XLIX.* [d] *I.* [e] *XV.* [f] *LXXVIII* [g] *Ax.* 5.

Note. From the two Preceding Propoſitions it appears, that figures inſcrib'd and conſcrib'd, if the number of their ſides be doubled, do approach the Circle by more than half the ſpace that was before left.

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A Circle contains more ſpace, than any other figure that is equal to a Circle.

FOr a ◯ is = [h] ∟ Angled △ whoſe baſe is the circumference of the ◯, and height, the Radius A B. But the Square C D (which we ſuppoſe = to the ◯) is [i] = to ∟ Angl'd △, whoſe baſe indeed, is the ſame with the former, (becauſe the compaſs of the Square is ſuppoſed = circumference of the ◯:) but the height is the Perpendicular A E. *Q. E. D.*

[h] *XCIX.* [i] *XCVIII.*

CHAP. IV. PART I.

Of the Power of Lines.

The Power *of Right-lines, is when they are so plac'd as to comprehend the greatest Space; which is when they are set at Right-angles. Therefore a* □ *and* □ *are call'd the* Power *of Lines.* Note, *two letters set together* AB *or* AE, *(or with a Note of Multiplication between them,* A × B, A × E,*) signifie a* □ *made of those two Lines. And* A A, *or* A × A, *or* A q, *or* QA, *signifies the Square of the Line* A.

Theorem CIII.

A Rectangle [Z B] *of two whole Lines* [ZB] *is* = *to the Rectangles* [AB + EB] *that are made of one whole* [B] *and of the parts* [A, E] *of the other.*

IF Z = A + E } For to = bases (*viz.* then ZB = AB + EB } Z, and A + E) there is
[k] *XXXIV.* added an = height (B) [k] therefore the Rectangles are =. *Q. E. D.*

Scholium.

Scholium.

The Rectangle of the whole Lines [Z B] *is also = to the Rectangles of the parts of each Line.* [*viz.* C, D. A, E.] *Fig. 11.*

FOr, $ZB^{l} = \left\{\begin{matrix} AB = \\ + \\ EB = \end{matrix}\right\}^{l} \left\{\begin{matrix} AC + AD \\ EC + ED \end{matrix}\right\}^{m}$ therefore, ZB = AC + AD + EC + ED. *Q. E. D.*

[l] *Pre.* [m] *Ax. 6.*

Theorem CIV.

A Rectangle [Z A] *of the whole, with one of its parts,* [A] *is = to the* □ *of the same part,* [A] *+ to the* □ *of the Parts* [A, E] *together.*

FOr, ZA[n] = AA (*i.e.* Aq) + Æ. For the same height, A is added to [o] = bases Z and A, E: Therefore the □ ZA is = AA + AE. *Q. E. D.*

[n] *CII.* [o] *Ax. 5.*

Theorem CV.

The Q *of the whole is* = Q *of the parts,* + *to the Rectangles made of the same parts, viz.* $Zq = Aq + Eq + 2Æ$.

p *CIII.*
q *Pre.*

$$\text{For } ZZ^{p} = \left\{\begin{array}{l} ZE^{q} = \\ + \\ ZA^{q} = \end{array}\right\} \begin{array}{l} EE\ (i.e.\ Eq) + EA \\ \\ AA\ (i.e.\ Aq) + AE \end{array}\Bigg\}$$

that is, $ZZ = Eq + Aq + 2Æ$. *Q. E. D.*

Theorem CVI.

Fig. Preced.

$$Zq + Eq = 2ZE + Aq.$$

r *CIII.*
s *CIV.*

$$\text{For } Zq^{r} = \left\{\begin{array}{l} ZE \qquad\qquad s = \\ + \\ (ZA^{s} =)Aq + (Æ \end{array}\right\} ZE - Eq$$

Therefore $Zq + Eq = ZE + Aq + ZE$. *i.e.* $2ZE + Aq$. *Q. E. D.* In numbers thus, $36 + 4 = 12 + 16 + 12$. Supposing Z to be 6, A 4. E 2.

Where *Note*, that a Square, is a number Multiplied by it self; and a Reactangle, two numbers Multiplied by one another. The like instance in numbers might be given in the other Propositions.

Theorem

Theorem CVII.

The Q *of the whole* [Z] *and of either of its parts,* [E] *added to it, (as of one line) is* = 4 □s *made of the whole* [Z] *and that same part* [E;] + Q *of the other part* [A].

Viz. Q, Z + E = 4ZE + Aq.
For, $QZ+E^2 = \underbrace{(Zq+Eq)}_{u} + 2ZE$. *Q.E.D.* [t] *CV.*
$2ZE+Aq$ [u] *Pre.*

Theorem CVIII.

The Q *of the half,* [½ Z] *is* = □ *of the unequal parts,* [A + E, *and* B] + Q *of the intermediate part* [E].

Viz. Aq (*i.e.* Q, ½ Z) = A + E × B + Eq.
For, A + E × A — E (*i.e.* B) [w] = (Aq — Æ + EA — Eq *i.e.*) Aq — Eq. Therefore, A + E × B + Eq = Aq. *Q.E.D.* [w] *Sch. CIII.*

Theorem CIX.

The Qs of the unequal parts [Z *and* B] *are equal* 2 *Qs of the half* [A] *and of the intermediate part* [E].

Viz. $Zq + Bq = 2Aq + 2Eq$.

For Zq (*i.e.* $Q.A+E$) $^{x} = Aq + 2Æ + Eq$.

x *CV.*

And, Bq (*i.e.* $Q.A-E$) $^{x} = Aq - 2Æ + Eq$.

Therefore $Zq + Bq = 2Aq + 2Eq$. *Q. E. D.*

For $+2Æ$ and $-2Æ$ deſtroy one another.

Theorem CX.

The Q of the whole [A] *with the piece added,* [E] *and of the piece added, is double to the Q of the half,* $\frac{1}{2}$A, *and of the half with the piece added* $\frac{1}{2}$A+E.

y *CV.* z *Ax.* 5. a *LXXX.*

Viz. $Zq + Eq = 2Q.\ \frac{1}{2}A + 2Q.\ \frac{1}{2}A + E$

For,

$Aq^{z} = \frac{1}{2}Aq^{a} = 2Q.\frac{1}{2}A$

$+\ \frac{1}{2}Aq^{a} = 2Q.\frac{1}{2}A$

$Zq^{y} = Aq^{z} + 2Æ^{z} + 2Eq = 2Q.\frac{1}{2}A + 4\frac{1}{2}Æ + 2Eq = 2Q.\frac{1}{2}A + E$ *Q. E. D.*

Theorem

Theorem CXI.

The Q *of the half with the piece added,* [E,] *is* = *to the* ▭ *of the whole,* [A] *and the piece added,* + Qs *of the half, and of the piece added.*

Viz. $Q\frac{1}{2}A + E$ [b] $= \underbrace{Æ}_{2\frac{1}{2}Æ} + Q\frac{1}{2}A + Eq.$ [c] ||

Q. E. D.

[b] *C V.*
[c] *Ax.* 5.

CHAP. IV. PART II.

Of the Power of Proportional Lines.

Theorem CXII.

In Proportional Lines [A . B :: C . D] *the* ▭ *of the extreams* [A D] *is* = ▭ *of the middle* [BC.]

DRaw out {T F and H E} to G. [d] Because the ▭ AB is :: CD ([e] since A . B :: C . D) [f] therefore they have the same Diameter LG: [g] therefore

[d] *Def.*
[e] *Hyp.*
[f] *LXXXIX*
[g] *XXX.*

fore the Triangles $\left\{\begin{matrix} LGH = LGI \\ LKA = LKB \\ KGE = KGF \end{matrix}\right\}$ therefore (the = Triangles being omitted) the □, KH = KI, and the common DC being added
[h] *Ax.* 8. to these, $DC + KH$ [h] $= DC + KI$, that is, AD = BC. *Q. E. D.*

Scholium.

In ∴ A, B, C. The □ of the extreams is
4. 6. 9. = to the □ of the middle.
viz. AC = Bq. *Note,* This Propoſition
36 = 36. is call'd the **Catholic Theorem** and is the Foundation of the *Golden Rule* in Arithmetick.

Theorem CXIII.

In a Rectangled Triangle [h e i.] *the* Q. *of the ſubtendent* [h, i.] *is* = *to the* 2 Q's. *of the* 2 *ſides, viz.* A = B + C.

FRom the ∟ E, let fall the Pp. ED. Then,
[i] *LXXVI.* [i] HI, IE, ID ∴ alſo IH, HE, HD ∴
[k] *LXXXIII.* [k] Therefore, A. $\left\{\begin{matrix} B \\ C \end{matrix}\right\}$ :: HI. $\left\{\begin{matrix} ID \\ HD \end{matrix}\right\}$ But, [l] HI =
[l] *Ax.* 5. $\left\{\begin{matrix} ID \\ + \\ HD \end{matrix}\right\}$ [m] Therefore, A = $\left\{\begin{matrix} B \\ + \\ C \end{matrix}\right\}$. *Q. E. D.*
[m] *Ax.* 10.

Theorem

Theorem CXIV.

In Acute-angled Triangles, the Q. of one side [C,] *is less than the Q's of the 2 other* [Z, B] *(by 2 Z E.)*

Viz. $Bq + Zq = Cq + 2ZE$. Let fall the Pp. DE.

For, [n] $Dq + Eq + Zq$

[o] $Dq + Aq + 2ZE$

[n] $Cq + 2ZE$. *Q. E. D.*

n *Pre.*

o *CVI.*

Theorem CXV.

In Obtuse-angled Triangles, the Q. of the subtendent of the Obtuse-angle, is ⊏ *than the Qs of the 2 other sides (by 2 Æ.)*

DRaw out A, to E, and let fall the Pp. DE. That is, $Bq = Aq + Cq + 2Æ$.

For, [p] $Zq + Dq$

[q] $Aq + 2Æ + Eq + Dq$

[p] $Aq + 2Æ + Cq$. *Q. E. D.*

p *CXIII.*

q *CV.*

Theorem

Theorem CXVI.

In Triangles, that Angle is right, the Q. of the whose ſubtendent is = to the Q's of the other ſides.

FOr if the Angle H E I, were Acute or Obtuſe, the Q. of the ſubtendent, H I, would be [r] ⌐ or [r] ⌙, then the Qs. of the other ſides, contrary to the ſuppoſition: Therefore it is ∟. *Q. E. D.*

[r] *CXIV.*
[s] *CXV.*

Theorem CXVII.

All like Figures made upon the ſubtendent of an ∟, are = to thoſe made upon the other 2 ſides, viz. A=B+C.

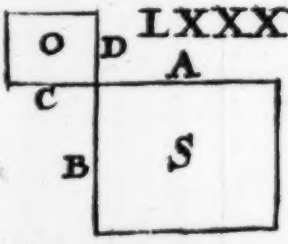
FOr, A, B, C, are to one another as the Qs [t] of their Homologous ſides, D E, D F, F E. But the Q. of D E is = [u] Q, D F + Q, F E. [w] Therefore the Figure A=B+C. *Q. E. D.*

[t] *XCI.*
[u] *CV.*
[w] *Ax.* 10.

Theorem

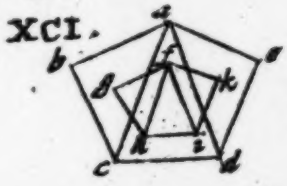
LXXXIV

LXXXV.

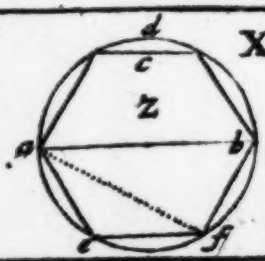


XC.

XCI.



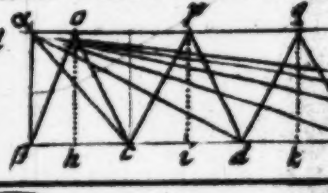
XCII.



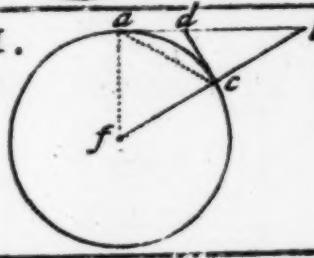
XCI

XC. VI.

XCVII. XCVIII

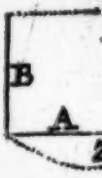


CI.



CII.

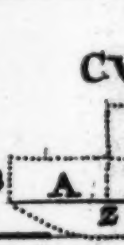
CII



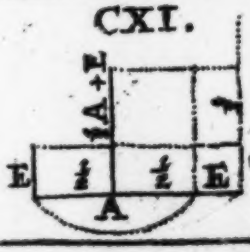
CV. CVI.

CVII.

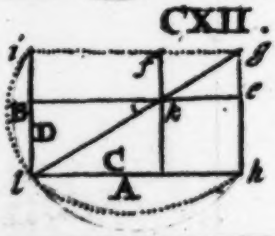
CV



CXI.



CXII.

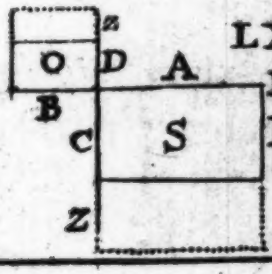


CXIII

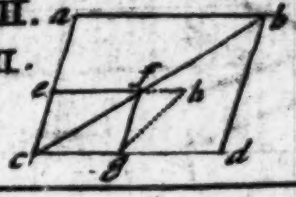
LXXXVI.

LXXXVII.

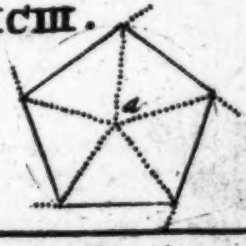
LXXXVIII.



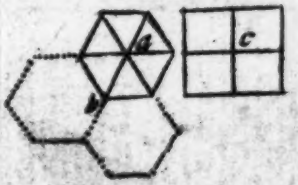
LXXXIX.



XCIII.

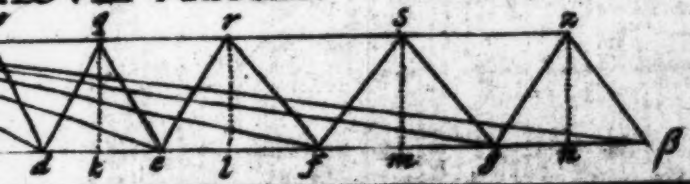


XCIV.

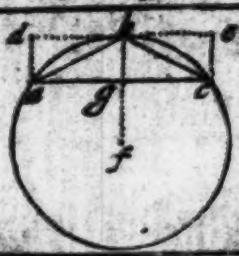


XCV.

XCVIII. XCIX.



C.

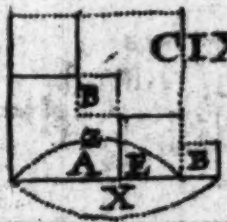


CIII.

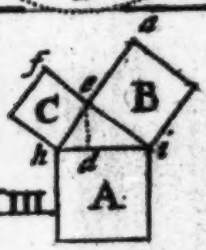
CIV.

CVIII.

CIX.



CX.



CXIII.

CXIV.

CXV.

Face to p. 88.

Theorem CXVIII.

The ſame thing is true of Semicircles.

FOr ○s are to one another as the [x] Q's of their Diameters, [y] therefore alſo Semicircles. But the Q. of the Diam. F H is = to both the Qs. of the Diam's. F G, G H, therefore the Semicircle B A = B D + C E. *Q. E. D.*

[x] *XCII.*
[y] *LXIII.*

Thorem CXIX.

The Squaring of the Lunes (or, half moons) of Hippocrates of Scio.

1. BEcauſe the Semicircle A B [z] = B D + C E; therefore, leaving out the common B and C, there remains the △, A = to the 2 Lunes, D + E.

[z] *Pre.* *F. CXVIII.*

2. When the △ is = Legg'd, the Lunes are equal, (the Diameters of the Semicircles being equal) and both together (D + E) are = [a] to the whole △, I K: Therefore either of them are = to ½ the △, *viz.* I or K.

[a] *CXIX. 1.* *Fig. CIX.*

3. But when the △ is ſcalene, it is as difficult to divide it, by the Line G Z, into 2 parts = to the reſpective Lunes D, E; as to find the Square of a Circle. Of which we ſpoke before in the Note on Theorem XCIX. *F. CXVIII.*

Theorem

Theorem CXX.

1. *Lines in a Circle,* [a b, d c,] *cut themſelves proportionally.* 2. *And their parts make* = □s, *viz.* □ BEA = CED.

b *LIV.*
c *III.*
d *LXXI.*
e *Def.* like.
f *CXII.*

1. FOr the Angle $\left\{\begin{matrix} A = D \\ C^{b} = B \\ DEB^{c} = AEC. \end{matrix}\right\}$ [d] Therefore the △ DBE is like AEC; and by [e] conſequence BE . CE :: ED . EA. 2. Therefore the □ BEA[f] = CED. *Q. E. D.*

Theorem CXXI.

Lines between Paralls. cut themſelves Proportionally, viz. A . B :: C . D.

g *LXXI.*
h *IV.*
i *III.*
k *Ax.* 13.

FOr the △ X, is[g] like Z, (becauſe the Angle I = [h] H, and G [h] = K, and the two head Angles are [i] =) and by conſequence: $\left\{\begin{matrix} E.A :: F.B \\ E.C :: F.D \end{matrix}\right\}$ therefore alternd, $\left\{\begin{matrix} E,F :: A.B, \\ C.D, \end{matrix}\right.$ [k] Therefore A . B :: C . D. *Q. E. D.*

Theorem

Theorem CXXII.

The □ *of the whole Secant with the part added, is* = *to the* Q. *of the Tangent, drawn from the end of the Secant.*

Viz. □ CAE = Q. AD. For the Angle $\left\{\begin{matrix}\text{ADE}^{l} = \text{ACD}\\ \text{A, is common:}\end{matrix}\right\}$ [m] Therefore AED = CDE: [n] Therefore the △ CAD is like AED; therefore, AC. AD. AE ∴ and by [o] consequence the □ CAE = Q. AD. *Q. E. D.*

[l] *LX.* [m] *XXII.* [n] *LXXI.* [o] *CXII.*

Theorem CXXIII.

Lines drawn from a point without, to the inward Circumf. of a O, *are to one another, as their parts without the* O. *viz.* A + B. C + D :: D. B.

FOr the △ HEK is [p] like IGK; ([q] becauſe the Angle K is common, and H is [r] = G;) therefore A + B. D :: C + D. B; and *Alternd.* A + B. C + D :: D. B. *Q. E. D.*

[p] *LXXI.* [q] *XXII.* [r] *LXX.*

Theorem

Theorem CXXIV.

If the Diameter of a O [a c] *be cut by infinite Perpendiculars, either in, at, or without the* O, *and a Secant* [a e] *be drawn, it will be,* AD . AC :: AB . AE

f *LXXI.* t *XXII.* u *Hyp.* w *LVI.*

BEcause the △s EAC and DAB are f like; (t for the Angle ECA u = ∟ w = ADB, and A is common,) therefore AE . AC :: AB . AD, and *Alterned Inverted,* AD . AC :: AB . AE. *Q. E. D.*

Scholium.

IN the second case (*viz.* at the ◯) AC, being the same with AB, AD, AB . AE are ÷; and by consequence the Diameter [AB] is a middle Proportional between the whole Secant [AE,] and the internal part of it [AD].

Theorem CXXV.

A Chord [a d] *dividing equally an Angle* [b a c] *in a Segment is (it self, and its lesser part) reciprocally proportional to the sides of the Angle, viz.* AC . AE :: AD . AB.

x *LXXI.* y *XXII.* z *Hyp.* a *LVI.* b *Def.* 54.

BEcause the △s, ABD and AEC, are x like; (for y the Angles at A are z =, and D a = C;) b therefore AC . AE :: AD . AB. *Q. E. D.*

Theorem

Theorem CXXVI.

The □ Circumscrib'd, (or the □ of the Diameter) is double to the □ inscrib'd, in a ○, viz. EFq = 2ECq.

FOr, EFq (*i.e.* AB) = [c] CFq + CEq (the Angle ECF, [d] being ∟ and EC = CF: Therefore EFq is double to either of them, CFq, or CEq, *i.e.* CD. *Q. E. D.*

[c] *CXIII.*
[d] *LVI.*
[e] *Def. of □.*

Theorem CXXVII.

A Trapezium (that is, an irregular *four sided figure) being inscrib'd in a Circle: The □ of the diagonals* [ACDB] *is = to the 2 □'s of the opposite sides; viz.* DC×AB, DA×CB.

TAke the ∠ADE = BDC. Therefore the △s DCE and DAB, also DAE and DBC are [f] :: (For ∠ $\begin{cases} \text{DCA} [g] = \text{DBA}, \\ \text{ADB} [h] = \text{EDC}. \end{cases}$ ∠EDB being common, and ADE being [i] = BDC: Also ∠ $\begin{cases} \text{DAE (that is DAC)} [i] = \text{DBC}. \\ \text{ADE} [h] = \text{BDC}. \end{cases}$)

[f] *LXXI. & XXII.*
[g] *Ax. 8.*
[h] *Const.*
[i] *LIV.*

[k] *Def.* 45. And by [k] consequence $\begin{cases} DC.CE :: DB.BA. \\ DA.AE :: DB.BC. \end{cases}$

[l] *CXII.* Therefore [l] $\square \begin{cases} DC, BA = CE, DB. \\ BA, BC = AE, DB. \end{cases}$

[m] *CIII.* Therefore $\square$ DC, BA + DA, BC = (DB, CE + DB, AE [m] =) DB, AC. *Q. E. D.*

CHAP. V.

Of Surfaces.

Theorem CXXVIII.

No part of a Right-Line [b c] *can be out of a Plane, when any part of it, is in it: That is, no Right-Line can lye upon two Planes.*

For if you say, that ABC, is part in the Plane, and part out, then by drawing the Line AC, AC will be [n] shorter than ABC. Contrary to *Def.* 4.

[n] *XI.*

Theorem

Theorem CXXIX.

1. *Every Right-lin'd* △ *is in the ſame Plane.*
2. *And alſo* 2 *Right Lines croſſing one another.*

1. FOr a [o] △ is a Plane Surface, comprehended within three Lines. [o] *Def.* 13.

2. If therefore, AB, CB are in the ſame Plane, AE, CD, [p] will be alſo in the ſame Plane. [p] *Pre.*

Theorem CXXX.

Parallel Right-Lines [ab, cd] *are in the ſame Plane.*

DRaw the Right-Lines AD, CB. The [q] △ CED is in one Plane; and alſo the △ AEB: But DE and EA are in the ſame Plane: Therefore both the △s are in the ſame Plane; and by conſequence their baſes, AB, CD. *Q. E. D.* [q] *Pre.* 1. [r] *Pre.* 2.

Theorem CXXXI.

The Interſection [a b] *of two Planes, is a Right-Line.*

IF A B, be not right, draw the right A G B in the Plane C D; and A H B in the Plane E F; that is, two Right-Lines between the ſame points. Contrary to *Def.* 4.

Theorem CXXXII.

The Interſections [a b, C D] *of two Parallel Planes, by a third Plane, are Parallel Lines.*

IF not, let A B, C D, meet in I: Therefore the Planes E F, G H, ſhall meet there alſo, [CXXVIII] (becauſe A B I, and C D I are in the ſame Planes) Contrary to *Def.* 65.

Theorem

Theorem CXXXIII.

A Line [dc] *that is Pp. to two Lines* [t] *croſſing*, [ac, bc] *is alſo Pp.* [u] *to their Plane* [af.]

[t] *CXXIX.* 2
[u] *Def.* 50.

[F] not, let DE be Pp. to the Plane AF, and from the point E, draw the Right-Lines EA, EC, to which, the ſaid DE ought to be Pp. by *Def.* 60.

Now, $\square AD^{a} = \begin{cases} AC^{b} = \begin{cases} AE \\ + \\ EC \end{cases} \\ + \\ DC^{b} = \begin{cases} ED \\ + \\ EC \end{cases} \end{cases}$

[a] CXIII. becauſe the Angle ACD, is $\perp$ by Hyp. [b] becauſe the Angles AEC, dec, ought to be $\perp$ by *Def.* 60.]

Therefore the $\square AD \sqsubset \square s. AE + ED$: Therefore the Angle DEA, cannot be $\perp$. [CXIII.] (the ſame reaſon will hold, if the Line DE, touches the Plane any where but in C) and by conſequence not DE, but DC [w] is Pp. to the Plane. *Q. E. D.*

[w] *CXVI.*

Theorem CXXXIV.

Three Lines which receive the ſame Pp. [E A] *are in the ſame Plane.*

IF not, let A B, be in another Plane [*viz.* E F] cutting the Plane G H, in A F: Since therefore, E A is Pp. [x] to A C, A D, it will alſo be Pp. to their Plane, G H: Therefore the Angle E A F, [y] is ⌊, as is alſo [x] E A B, and by conſequence E A F is [z] = E A B; that is, the part = to the whole. *Q. E. A.*

[x] *Hyp.*
[y] *Def.* 60.
[z] *Ax.* 2.

Theorem CXXXV.

If of two Parallels, one [a b] *be Pp. to a Plane, the other* [c d] *is alſo.*

IF not, let E D be Pp. This will be in the Plane [a] D F. Now, the Angle E D B [b] (⌊ [c] = A B I) = [d] C D B, *viz.* part = whole. *Q. E. A.*

[a] *CXXIX.*
[b] *Def.* 60.
[c] *Ax.* 2. *and Hyp.*
[d] *IV.*

Theorem

Theorem CXXXVI.

One only Pp. [cd] *can be rais'd from one point in a Plane.*

IF it is poſſible, let CE be alſo Pp. and through theſe two CE, CD, let a Plane be [e] drawn; cutting the other Plane in ACB, the Angles ACE, ACD, muſt be both [f] ∟; and by conſequence =, *viz.* part = whole. *Q. E. A.*

[e] *CXXIX.* 2

[f] *Def.* 60.

Theorem CXXXVII.

A Pp. [fe] *to one Plane,* [ab] *is Pp. to its Parallel Plane.*

DRaw a Plane through FLE, cutting the others in LH and EG, theſe interſections will be [g] Parallel. Now the Angle GEF, is ∟ [h], and = [i] HLF: (And the like, if the Plane paſs through EM, LO,) [k] therefore EF is Pp. to the Plane, CD. *Q. E. D.*

[g] *CXXXII.*

[h] *Hyp.* [i] *IV.*

[k] *Def.* 60.

Theorem CXXXVIII.

Planes are Parallel that receive the ſame Pp.

[l] *XVI.* FOr ſince the Angle GEF = HLF, [l] therefore the Line HL is Parallel EG, (and the like of ME to OL, *&c.*) whereſoever the interſection can happen; therefore the Plane AB is Parallel to CD. *Q. E. D.*

Theorem CXXXIX.

If two croſſing Lines, [bac] *are Parallel to two croſſing Lines* [edf] *in another Plane, their Planes alſo will be Parallel.*

[m] *Hyp.*
[n] *Def.* 60.
[o] *VI.*
[p] *conſtr.*
[q] *CXXXIII*
[r] *Pre.*

DRaw AG Pp. to the Plane EF, and HGI, Parallel (EDF, [m] Parallel) BAC: Therefore the Angles HGA, IGA [n] are ∟ [o] = CAG, BAG: Therefore AG is Pp. to the Plane [p] EF and [q] BC: And by [r] conſequence the Planes BC, EF are Parallels.

Theorem

Theorem CXL.

Two meeting Lines [b a c] *Parallel to two* [e d f] *in another Plane, contain an* = *Ang. with them,* viz. *Ang.* A=D.

TAke AB=DE, and AC=DF, and joyn BE, AD, CF: Since AB, ED, and AC, DF, are [r] Parallel and [t] =; BE is alſo [u] = and Parallel, (to AD [u] = and Parallel) [w] CF. Therefore BC [u] = EF; and by conſequence the △BAC [x] = EDF, and the Angle A = D. *Q. E. D.*

[r] *Hyp.* [t] *Conſtr.* [u] *XXX.* [w] *VIII.* [x] *XVI.*

Theorem CXLI.

Parallel Lines [a e, b f] *are cut proportionally by Parallel Planes.*

Viz. AC . CE :: BD . DF: Becauſe the interſections AB, CD, EF, are [y] parallels; [z] therefore $\frac{BD}{DF} = \left(\frac{BZ}{ZE} =\right) \frac{AC}{CE}$. *Q. E. D.*

[y] *CXXXII* [z] *LXVII.*

Part II.

PART. II.

Of a Solid Angle.

Theorem CXLII.

Of three Angles which are neceſſary to compoſe a Solid Angle [a] *any two, are* ⌐ *than the third.*

IF three Angles are =, it is evident: But if one be biggeſt, *viz.* BAC, take BAE = BAD; joyn BC, and by the Lines BD, CD, take off AD = BE: Becauſe, AD = AE, and AB is common, and the Angle BAE [a] = BAD: [b] Therefore BD = BE; but BD + DC [c] ⌐ BC: Therefore (BD being [a] = BE) DC [d] ⌐ EC; and by conſequence the Angle DAC [e] ⌐ EAC; therefore (BAD being [a] = BAE) BAD + BAC [d] ⌐ BAE + EAC, *i. e.* BAC. *Q. E. D*

[a] *conſtr.*
[b] *XVII.*
[c] *XI.*
[d] *Ax. 9.*
[e] *XIX.*

Theorem CXLIII.

A Solid Angle [a] *is made up of leſs than Four* ⌊ *Angles.*

FOr, the 6 Angles at B, C, and D, + to the 3 at A, are [f] = 6⌊; but the 6, at B C,

[f] *X.*

C, and D, are [g] ⊏ than (the 3 B, C, D, in the base of the Pyramid [f] =) 2 ⌊ : Therefore the 3 at A, are ⌐ 4 ⌊. *Q. E. D.*

[g] *Pro.*

The reason is this, since the Four Angles made by 2 Lines crossing, (as at Z) are [h] = 4 ⌊ ; The Plane [i] in which they are cannot be so folded (in the Lines ZF, ZG, ZE, ZH) as to make a Solid Angle at Z, unless something be left out from one of the Angles, as FZS.

[h] *I.*

[i] *CXXIX.*

CHAP. VI.

Of Bodies, or Solids.

Theorem CXLIV.

The opposite Planes [a c, d b] *of a Parallelepipedon* (*Ppp.*) *are like and equal.*

AC, DB, are [k] Parallel: Therefore the intersections AF, DE are [l] Parallel; and AFDE is a [m] Pgrm. by consequence AF [n] = DE. After the same manner it may be prov'd that all the other opposite Lines are Parallel and =: Therefore the Angle FAH = EDG, and AFC = DEB. Lastly, the Plane AC is [o] = and [p] like DB, and so of the rest. *Q.E.D.*

[k] *Def.*

[l] *CXXXII.*

[m] *Def.* 20.

[n] *XXX.*

[o] *XXXIV.*

[p] *Def.*

Theorem

Theorem CXLV.

A Ppp. is divided in the middle, by a Plane [ah] *that passes through the Diameters* [ea, hm.] *of two opposite Planes.*

q *Pre.* r *XXX.* ALL the opposite Pgrs. are q =: The Δ AED r = AEF, and LMH r = MHG, the Plane AH is common, therefore the Part or Prism AL = AG. *Q. E. D.*

Theorem CXLVI.

Ppps. [ZX *and* ZS] *are* = *which have the same base and height.*

(*Note*, This Figure would be best comprehended, if it were cut out in Cork, or the like matter.)

1. LEt them fall between the same Parallels, KF, and DA: Therefore the Prism (EDH, *i. e.*) S + O = O + X. [And (the common O, being left out) S = X, and (Z being added) S + Z = X + Z. *Q. E. D.*]

s *XXIV.* t *XXX.* u *IV.* w *CXLIV.* x *Hyp.* y *XXXIV.* z *Ax.* 8. a *Def.* Because, the Plane 1. EPH s = FRA, EH being t = FA, EP t = FR Ang. P = u R, 2. The Planes opposite to these, are = by the same reason. 3. DE w = GF, and LE y = CF, being between the same x Parallels, and the bases KE t = MF. 4. GA = DH; because DR w = (KF w =) LA: Therefore DR + GH z = LA + GH. a Therefore S + O = O + X. 2.

2. If a Ppp. of the ſame baſe and height with Z X, ſhould not fall between the ſame Parallels; It will however fall between the ſame Parallels with S Z; and being, as before, = prov'd to it, will be [b] = to Z X. [b] *Ax. 6.*

Theorem CXLVII.

Ppps. [S Z, O] *are* = *which have an* = *and like baſe, and the ſame height.*

UPon the baſe C A B, ſuppoſe a Ppp. fram'd Z X, of the ſame height, and like to O. Then, O = Z X. (for the ſeveral ſides will be prov'd = by XXXIV. and *Like*, by conſtruction) which Z X [c] = S Z; therefore O [d] = S Z. *Q. E. D.* [c] *Prec.* [d] *Ax. 6.*

Theorem CXLVIII.

Ppps. [a b, a d.] *of the ſame height, are to one another as their baſes.*

LEt the baſe C G, be made = C E; therefore the Ppp. S = A H: Now Let the Plane B F, (being [e] Parallel G H and A C) be mov'd Parallely from A C to G H. It ſhall at the ſame time diſpatch the baſe G C, and the Ppp. A H: And by conſequence how much ſoever it has taken away at any time from the baſe, it will have taken away a *Like* part from the Ppp. ſo that, as the baſe G C is (for inſtance) to its part G F:: Ppp. A H (= S X.) *Q. E. D.* [e] *Def. Ppp.*

Theorem

Theorem CXLIX.

Equal Ppp's. [S = XO] *have their bases and height, reciprocally proportional, viz.* AB . CD :: MK . AI.

Def. 46.

TAke the Ppp. X, of the same height as S, that is, ML = AI. Then, $\frac{AB}{CD}$ [f] = $\left(\frac{S}{X}\right.$ [g] = $\frac{O+X}{X}$ [f] = $\frac{DK}{DL}$ [h] = $\frac{MK}{ML}$; that [g] is $\left.\right)\frac{MK}{AI}$ (because ML = AI, Constr.) *Q. E. D.*

[f] *Pre.* [g] *Ax.* 10. *because* O + X = S. *Hypo.* [h] *LXXX.*

Theorem CL.

Those Ppps. [S = XO] *are equal, whose bases and heights are* [a] *reciprocally proportional, viz.* AE . FC :: DN . BE

[a] *Def.* 46.

FOr, Ppp. $\frac{S}{X}$ [i] = $\left(\right.$ *bas.* $\frac{AE}{FC}$ [k] = *height* $\frac{DN}{BE}$, *i.e.* MN [l] = *side* $\frac{DF}{HF}$ [m] = *Pgr.* $\frac{DK}{HK}$ [i] = $\left.\right)\frac{XO}{X}$. Therefore S [n] = XO. *Q. E. D.*

[i] *ex LVIII. with LXI.* [k] *Sup.* [l] *LXVII.* [m] *LXXX.* [n] *Ax.* 10.

Theorem

CXVI

CXVII

CXVIII

CXXII

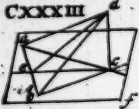
CXXIII

CXXVII

CXXVIII

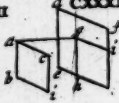
CXXXII

CXXXIII

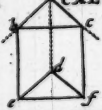


CXXXVII. CXXXVIII

CXXXIX

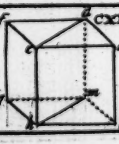


CXL



CXLIV

CXLV



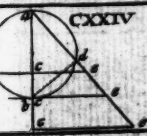
OREMS

CXIX

CXX

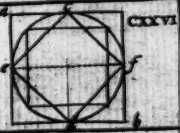
CXXI

CXXIV



CXXV

CXXVI



CXXIX

CXXX

CXXXI

CXXXIV

CXXXV

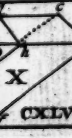
CXXXVI

CXLI

CXLII

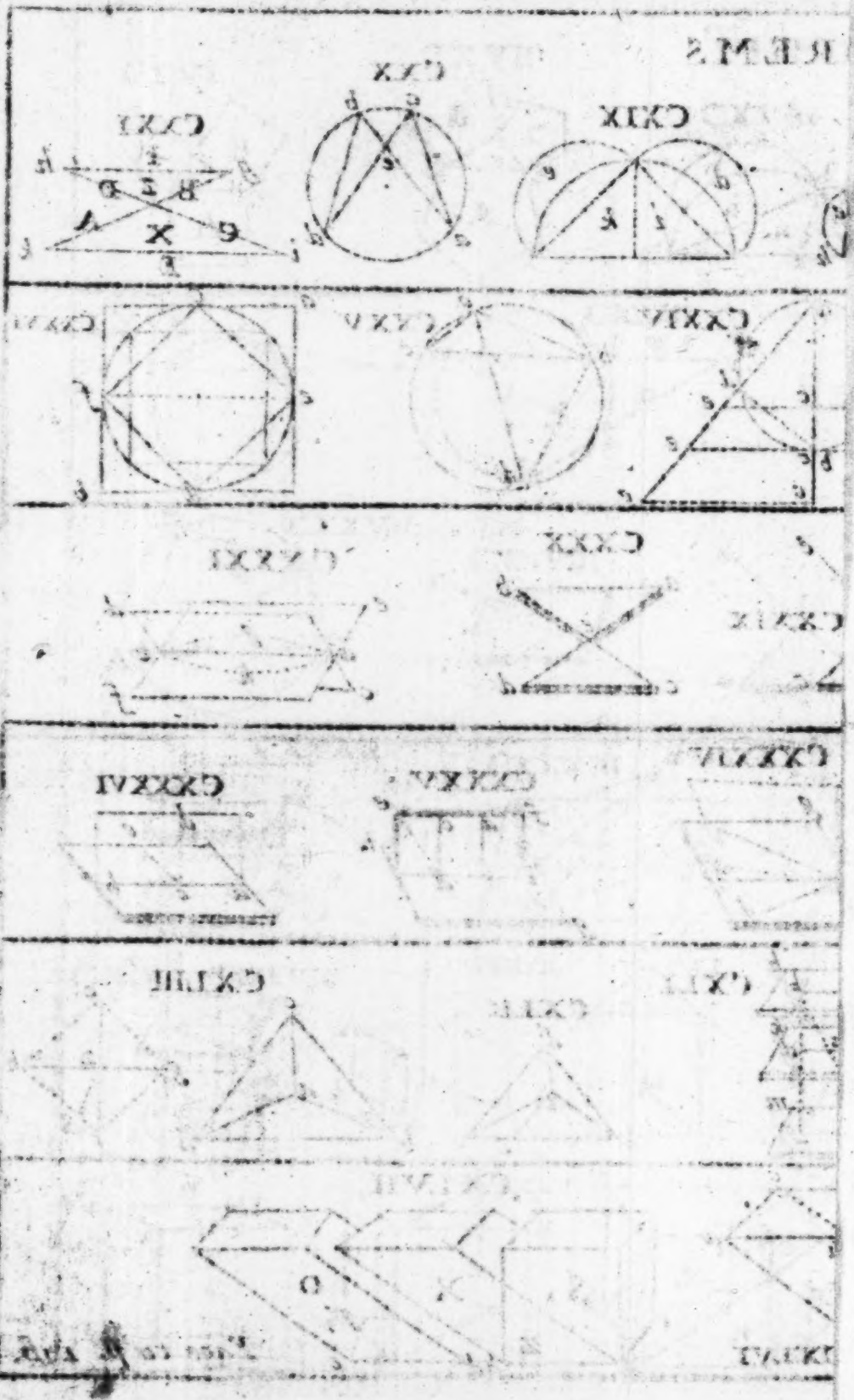
CXLIII

CXLVI



CXLVII

Page to p. 106.



Theorem CLI.

Like Ppps. [S :: O] *are in a triplicate reaſon of their homologous ſides, viz.* S . O :: AE . ED. *thrice.*

BEcauſe the $\angle DEF^{o} = AEH$, therefore [o] *Def.* 45.
[p] AE, BD may be plac'd in a Right Line, [p] *III.*
fill up the void ſpace with X and Z: Now,

$$\left.\begin{matrix} \frac{S}{X} \overset{q}{=} \left(\frac{AH}{EI} \overset{r}{=}\right) \frac{AE}{ED} \\ \frac{}{Z} \overset{r}{=} \frac{LE}{IN} \\ \frac{}{O} \overset{r}{=} \frac{HE}{EF} \end{matrix}\right\} \text{Alſo, } \frac{AE}{ED} = \frac{LE}{IN} = \frac{HE}{EF}. \left\{\begin{matrix} \text{For } \frac{AE}{LE} \overset{f}{=} \frac{ED}{(FC, i.e.)\,IN} \\ \text{and } \textit{Alterned } \frac{AE}{ED} = \frac{LE}{IN}. \end{matrix}\right.$$

[q] *CXLVIII*
[r] *LXXX.*
[f] *Def.* 45.

Again, $\frac{LE}{HE} = \frac{(FC\,i.e.)\,IN}{EF}$, and *Alterned* $\frac{LE}{IN} = \frac{HE}{EF}$) So then it appears that, $\frac{S}{X} = \left(\frac{AE}{ED} = \frac{LE}{IN} = \right) \frac{X}{Z}$ $\left(\frac{LE}{IN} = \frac{HE}{EF} = \right) \frac{Z}{O}$: Wherefore $\frac{S}{X} = \frac{X}{Z}$ [t] $=$ [t] *Ax.* 10.
; that is, S, X, Z, O $\div\div$ and by conſequence
$= \left(\frac{S}{X}\right.$ thrice, which $\left.\frac{S}{X} = \right) \frac{AE}{ED}$ thrice. [u] *Def.* 42.
E. D.

Scholium,

Scholium to this, and the former Theo.

BEcause Ppps. are extended in length, breadth and depth; therefore the the reason of one Ppp. to another, may be known, we are to take the reasons of all the dimensions together, viz. *if a Ppp.* A, *be twice as broad, three times as long, and four times as high as* B; *then* A *is twice* 3 *times* (*i. e.* six times) 4 *times* (*i. e.* 24 times) *as great as* B. *But if the Ppps. are* like: *Then the reasons of their length, breadth and depth are the same*[a]: *So that if* A *be twice longer, it must be also twice broader and twice higher than* B. *That is, twice twice* (*i. e.* 4 times) *twice* (*i. e.* 8 times) *greater than* B. *So that the same reason is taken* 3 *times; and is thereupon called* Triplicate Reason.

[a] Def. 45.

Theorem

Theorem CLII.

Like Ppps. are to be one another, as the Cubes of their Homologous *sides.*

BEcauſe all Cubes are *like* Ppps. and by conſequence are to one another in a Triplicate Reaſon of their ſides (by Prec. Theor.) But like Ppps. are in the ſame [a] reaſon. Therefore they are to one another as the Cubes rais'd upon their Homologous ſides. *Q. E. D.*

[a] *CLI.*

Theorem CLIII.

WHat has been ſaid in the foregoing Propoſitions concerning Ppps. does agree alſo to Priſms. (as being the [a] halves of a Pp) Provided that their oppoſite Parall. Planes, if they are not △s, be reſolv'd into △s.

[a] *CXLV.*

Theorem CLIV.

They agree alſo to Cylinders. For Example. (according to CXLVI) *If Cylinders have the ſame baſe* [G B] *and equal height then they are equal.* viz. A B = B C.

IF not; let A B be ⊏; and let there be inſcribed in it a Priſm of many ſides (as A D F E) ⊏ alſo then the Cyl. B C. (Becauſe by multiplying the ſides of the Priſm, you may approach by infinite degrees nearer and nearer to the Cyl. in which it is inſcrib'd. Beſides that, though the degree by which the Cyl. B C. is leſs than B A, be ſuppoſed to be never ſo ſmall, yet it is ſtated and fixt, and may not (after it is given) be alter'd. Whereas our Approaches in the Priſm may be made to Infinity.) Then upon the ſame baſe, G F, let there be inſcrib'd a Priſm in the Cyl. B C; Theſe Priſms will be [a] equal, that is, the Priſm H C F (= D A F) ⊏ [b] Cyl. B C. A part greater then the whole. *Q.E.A.*

[a] *Pre. & CXLVI.*
[b] *Conſtr.*

Theorem

Theorem CLV.

A Plane [l m n] *cutting a Pyramid* [b d] *Parall. to the baſe* [a b c] *makes a figure* like *the whole.*

FOr the Ang's at the top are common, and thoſe at the baſes, may be proved = by Theor. IV. Then, that the ſides about theſe reſpective <s are proportional, appears thus; A B . L N : : [a] A D . L D; and alterned; A B . A D : : L N . L D and ſo of the others. Wherefore the figures are [b] like. *Q. E. D.*

[a] *LXIX.*

[b] *Def.* 45.

Theorem CLVI.

The Sections [l m n o p r] *of two Pyramids (whoſe baſes, and heights are =) made by a plane* [l n r p] *Parall. to both their baſes, are equal.*

BEcauſe the baſes, A B C, E F G are : : [a] the Sections L M N, O P R; therefore $\frac{ABC}{LMN}$ and $\frac{EFG}{OPR}$ in a duplicate [b] reaſon of their Homologous ſides, but A B C is = [c] E F G. Therefore, L M N = [d] O P R. *Q. E. D.*

[a] *Prec.*

[b] *XCI.*

[c] *Sup.*

[d] *Ax.* 10.

Theorem CLVII.

All Pyramids are equal, which have equal baſes and height.

FOr inſtance C D = F H. If not, let C D be ⊏; and inſcribe in this a ſolid Segment, (made up of Priſms having *like* baſes and = height, L N M) ⊏ than the Pyram F H. (In which we argue, as before we did in Th. CLVII. becauſe, the height of the Priſms being leſſen'd, they may be infinitely multiplied, ſtill approaching toward the Pyram. in which they are inſcrib'd; till at laſt they may be made to come nearer to it than any other quantity that has been already given, as F H.) Then let there be inſcrib'd alſo in F H, as many Priſms (for there can be no bound to the number.) Both theſe Segments of Priſms ſhall be =; (for their number is =, *by Conſtruction*; Their baſes and heights may be equal, becauſe thoſe of the Pyramids in which they are inſcrib'd, are = *by Suppoſition*) that is, the Prz. in H F (= Prz. in D C) ⊏ [a] Pyram. H. F. *Q. E. D.*

[a] *Conſtr.*

Theorem

Theorem CLVIII.

Every triangular Prism *(that is, which has a* △ *for its baſe) may be divided into* 3 *equal Pyramids*; viz. EZD,A. ABC,D. AFC,D.

FOr the baſe ABC = [a] EFD; and the height is equal. [a] Therefore, ABC,D = [b] EFD,A = AFC,D. Becauſe the baſe AEF = [c], [a] AFC. and the height at D is common. *Q. E. D.*

[a] *Def.* 61.
[b] *Prec.*
[c] *XXX.*

Note, This Figure will be plain, if cut out in Cork.

Theorem CLIX.

WHat has been demonſtrated of *Priſms*, is true alſo of *Pyramids*, as being the third part of a *Priſm*. [a]

[a] *LXIII. & Prec.*

Theorem CLX.

What has been demonſtrated of Pyramids *(in the Precedent Theor.) agrees alſo to* Cones.

FOr inſtance. The Cone ABC = DEF; having = baſes and height. If not, let

A B C, be □; and let there be inſcribed in this a Pyram. A G H C, □ Cone D E F. (which, that it may be done, we prove by multiplying the ſides, as we have done before,) Then (becauſe the baſes of the Cones are = [a]) let there be inſcrib'd in the Cone D E F, a Pyram. E D K L M = [b] and : : A G H C. Now, the Pyram. E D K L = (Pyr. A G H C) □ [c] cone E D K. that is, a part □ whole. *Q. E. A.*

[a] *Sup.*
[b] *Prec.* being the ſame height.
[c] *Conſtr.*

Theorem CLXI.

All like *Bodies are in a Triplicate Reaſon of their Homologous ſides.*

BEcauſe they may be reſolv'd into Triangular Pyramids, equal in number, [a] and like. [b] But ſuch △r Pyramis: are in a [c] Triplicate Reaſon of their homol. ſides; therefore [d] alſo the Bodies. *Q. E. D.*

[a] *Def.* 45.
[b] *CLIX.*
[c] *CLIX.*
[d] *LXIII.*

Theorem CLXII.

A Sphere *is* = *a* Cone *whoſe perp. axis is the radius of the* Sphere, *and its baſe* = *the whole ſurface of the* Sphere.

THe ſame Demonſtration ſerves here as in Theor. XCVII. by ſhewing that all Polygons circum-

circumſcrib'd, or inſcrib'd in a *Sphere* are ⊏ or ⊐ than ſuch a *Cone*. Therefore the *Sphere* is = to ſuch a *Cone*. *Q. E. D.*

Theorem CLXIII.

Of all ſolid figures (having an = ſurface) The Sphere *is the greateſt.*

THis alſo is demonſtrated from the Precedent, by Theor. CII.

Theorem CLXIV.

Spheres are in a Triplicate Reaſon of their Diameters; *or, as the Cubes of their Diameter's* (*Theor.* CLI. CLII.) viz.

Sph. $\frac{ABC}{DEF} = \frac{AC}{DF}$ *thrice.*

IF not, let the Sphere $\frac{ABC}{DEF}$ be ⊏ $\frac{AC}{DF}$ thrice; and let there be inſcrib'd in the Sph. A B C a ſolid figure of many ſides, A B C G, ⊏ to the ſide D E F than A C to D F *thrice*. (ſince by continual doubling [a] the ſides of the body inſcrib'd, [a] *Theor.* C.

as AI, IB, &c. whereby the same line AC will always continue to be both the Diam. of the ○, and the side of half the Polygon, we may approach still nearer to the Sph. ABC, and by consequence, make it bigger than any other body, which is less than the Sphere.) Then let there be inscrib'd in the other Sphere a *like* body of many sides. These bodies, being resolv'd into Pyramids and so into △s. [b] will be in a Triplicate Reason of their homol. sides, AC, DF. *viz.* the [c] Body ABCG. DEFH :: AC. DF *thrice.* But $\frac{ABCG}{DEF} \sqsubset^{d} \frac{AC}{DF}$ *thrice.* Therefore the Sph. DEF, is ⊐ than the Body inscrib'd in it, DEFH. *Q. E. A.*

b *CLXI.*

c *Ax.* 14.

d *Constr.*

e *Ax.* 11.

Theorem CLXV.

There can be but 5 regular *Bodies, (which have all their sides, and all their Angles* =)

FOr no plane figures, joyn'd together, can make a solid Angle; except a △, □ and *Pentagon.* For a solid Ang. ought to consist of less [a] than 3 ∟s. and 3 plane ∠s are the fewest that can make up a solid one (as is naturally evident.) Now 3 ∠s of a Hexagon are = 4 ∟; and in all figures of more sides than a Hexicon. they exceed 4 ∟. Wherefore 3, 4, and 5 △'s

a *XLIII.*

△'s (for 6, make [b] 4 ∟s) 3 □, and 3 *Pentagons* (which make [c] $\frac{3}{10}$ of 4 ∟s) may all compose a solid Angle. Upon this account there can be but 5 *regular* Bodies. *Viz.*

[b] *XCIV*, 3.
[c] *XCIV.*

1. *A Tetraëdrum.* [d]	Whose Angle is made up of	3	Equilateral △s.
2. *An Octaëdrum.*		4	
3. *An Icosaëdrum.*		5	
4. *A Hexaëdrum.*		4	□s
5. *A Dodecaëdrum.*		3	*Pentagons.*

[d] *Def.* 74. *&c. Fig. CLXIV.*

If these, or *like* figures, be cut in Past-board according to the figures in the Type, and folded up, they will represent the foresaid 5 regular Bodies.

Theorem CLXVI.

It will not be amiss in conclusion of the Theorems, to add one Proposition that may serve as an Introduction to the Doctrine of Infinites. *viz. That* Infinites *may be* actually *number'd or measur'd.*

IF from the line A B you take (*suppose*) a fourth part toward A; [A C] and again towards B, 2 such parts, [D B] (*viz.* such a number of parts, less by 2, than the whole line was first supposed divided into) there will remain C D, one fourth of A B. If again, from this *remainder,*

der, you take as before, one part towards A, [C E] and 2 ſuch towards B, [F D] there will remain only E F one fourth of C D. And ſo if you continue to do to every remainder, there will always remain between the lines laſt taken, one fourth of the line from whence they were taken. From which fourth part, there may ſtill after the ſame manner be ſuppoſed to be taken two other ſuch lines on each ſide; but if this be done infinite times *actualy*, then there will nothing more remain (between) and ſo the continued diviſion on either ſide will come exactly to the point G; which cuts off a third part of the line A B. (*viz.* a part, of one Denomination leſs, than the whole line is firſt divided into.) Becauſe there was always taken away twice as much towards B as towards A. The total ſum, therefore, of all together that is taken away towards B, will be twice as much as the Total Sum towards A. And by conſequence, the *meeting* will be at ſuch a point [G] as cuts off G B double to G A. Which was the thing undertaken; viz. *to aſſign the preciſe meaſure of a line infinitely divided.*

But if it be inquir'd how this diviſion can be made infinite times *actually*; I anſwer; Let two points begin to move from A and B, at the ſame time; That at B, always moving as faſt again as that at A. It is certain they will at length meet; and (by the former Demonſtration) juſt at the point G. For the point B, will be at D, when A is at C. and B again at F, when A is at E. and ſo they will each paſs through the infinite Sub-diviſions before mentioned; and when they meet, will have divided the line A B infinite times *actually*.

The

The ſame may be ſaid, if one of the points be to overtake the other; as in the famous Sophiſm of *Zeno*, who argu'd that a Horſe would never overtake a Snail. For ſuppoſe the Horſe run ten times as faſt as the Snail, and the Snail were a Mile before, and both ſet out at the ſame time. When the Horſe has run the firſt Mile, the Snail will have got to the 10th part of the 2d Mile; when the Horſe has run this 10th part, the Snail will be got to the 10th of the next 10th part; (that is, a 100th part of the 2d Mile) and when the Horſe has got this, the Snail will be a 1000th part before him. And ſo in infinite Subdiviſions, when the Horſe has got the laſt part, the Snail will ſtill have got a part before him. On the contrary, it is naturally evident, as any axiom, that the Horſe will at length overtake the Snail; and (by conſequence) meaſure out thoſe infinite Sub-diviſions *actually*; The ſuppoſion of the Impoſſibility of which, is the ground of the Fallacy. Now the preciſe point of meeting may be determin'd, by a very eaſy Demonſtration. *viz. at the end of the 9th part of the 2d. Mile.* For ſince the Horſe runs 10 times as faſt as the Snail, the Horſe will run $\frac{1}{9}$ of a Mile, while the Snail runs $\frac{1}{9}$; and by conſequence they will come both at the ſame time to the end of the firſt 9th part of the 2d Mile. *Q. E. D.*

Theorem

PROBLEMS

Which purpose something to be done.
Demands, or Suppositions.

1. *That a right line may be drawn from any one point, to another.*
2. *That a right line may be continued at either end as far as we please.*
3. *That a Circle may be describ'd upon any Center, and at any distance, (or interval.)*

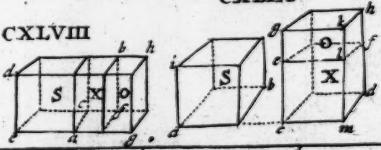
Problem I.

From a point given [a] *to draw a right line* [a c] *parall. to another* [b c].

ON the Cent. A, at any distance, (so as to cut the line B C) describe the Arch, D G E, Cent. D, same distance describe another Arch cutting the line given (B C) in G. Lastly, Cent. G same distance, cut the first Arch in E, the line

CXLIX

CXLVIII

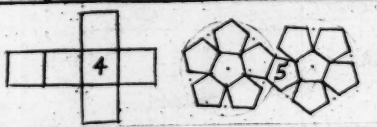


CLIV

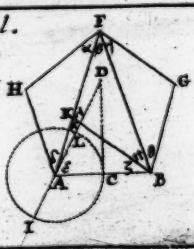
CIV

CLIX.
CLX.

CLXIV

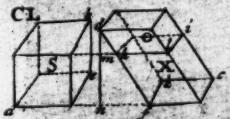


Probl.
XX.

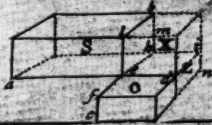


THEOREMS.

CL



CLI



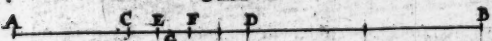
CLVI

CLVII

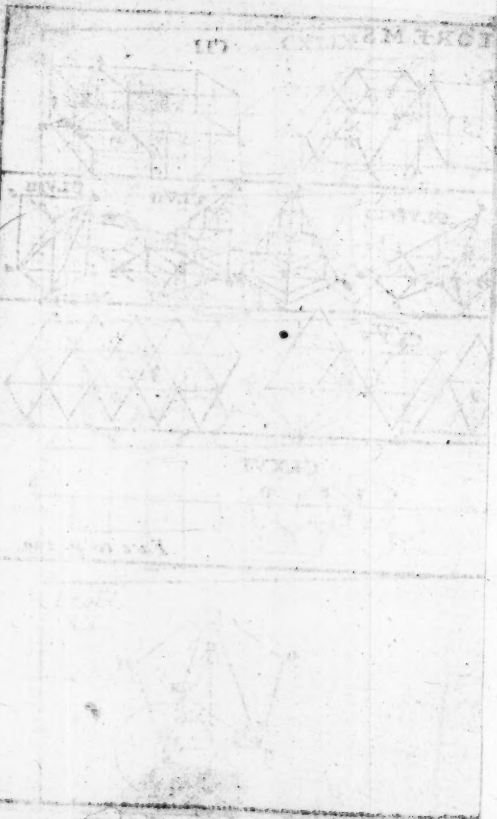
CLVIII

CLXV

CLXVI



Face to p. 120.



ne AE is parall. BC. For AD = [a] AE = [b] DG = [b] GE, and AG is common. Therefore in the △'s AEG and AGD, the ∠ GAE = [c] AGD. Therefore [d] AE is parall. to BC. *Q. E. F.) which was to be done.*

[a] *Radius's of the same* ○.
[b] *Rad's of* = ○s.
[c] *XIII.*
[d] *VI.*

Problem II.

From a point given, [a] *to draw a line,* [a e] *equal to a line given* D G.

FRom the point A, draw A E parall. [d] to D G, join the points A D. From the point G, draw G E parall. [d] to A D. The line A E is = D G, because in the Pgr. [e] D E, the oppoſite ſides [f] A E = D G. *Q. E. F.*

[d] *Preced.*
[e] *Def.* 20. *by Conſtr.*
[f] *XXX.*

Problem III.

From a line given, [a b] *to cut off a part* [a f] *equal to another given* [c d].

FRom the point A, draw A E = [g] C D, Cent. A. interval C D, (or A E) deſcribe the Arch E F. The line F A is = C D, becauſe F A = [h] (A E = [i]) C D. *Q. E. D.*

[g] *Prec.*
[h] *Radius's.*
[i] *Conſtr.*

Problem

Problem IV.

To make an Ang. [g] = *to an Ang. given* [b].

[k] *Prec.* FRom the line L F, take the part [k] L D = A B and D E = A C and E F = C B, then on the Cent. $\left\{\begin{matrix} D \\ E \end{matrix}\right\}$ interv. $\left\{\begin{matrix} DL \\ EF \end{matrix}\right\}$ describe Arches cutting in G, joyn G D, G E. The Ang. G is = B, (because the △ D G E is equal[m] sided to A B C.) *Q. E. F.*

[l] *XVI.* [m] *Constr.* [n] *Ax. 6.*

Problem V.

To divide an Ang. given, [b] *in two equal parts.*

CEnt. B, at any Interv. describe the Arch A C. Cent's. A and C, same Interv. descr. Arches crossing in D, joyn D A, D C, and draw D B, which divides the Ang. D in two = [o] parts. (For the △s B A D, B C D are equal sided, the sides B A D, B C D, being Radius's of = ○s and D B common, therefore the Ang. C D B = A D B.) *Q. E. F.*

[o] *XVI.*

Problem

Problem VI.

From a point given, [a *or* d] *to raiſe, or let fall a Perp.* [a d].

I. FRom each ſide of D take equals,[o] D B, D C. Cent. and interv. B, C, deſcribe Arches croſſing at A, joyn A D, which is the Perp. required. For A B is $=$ [p] A C, D B $=$ [q] D C, D A common. Therefore the Angles at D are $=$ [r], by conſeq. $\perp$ [s] and D A is perp. [t]

II. Cent. A, at any interv. deſcr. the Arch A C, joyn A B, A C, divide the $\angle$ A in the [u] middle, by the time A D which is the perp. [t] required. For the Angles at D may be proved $=$, as before. Therefore, &c. *Q. E. F.*

[o] *Prob. III.*
[p] *Radius's*
[q] *Conſtr.*
[r] *XVI.*
[s] *I.*
[t] *Def. 7.*
[u] *Pr. V.*

Problem VII.

To divide a line [a b] *in the middle.*

CEnt. A B, any interv. deſcribe Arches croſſing in D, let fall the perp. D C [w]. This divides A B in the middle. For A D $=$ [x] B D. D C, com. the Angles at D are $=$ [y]. Therefore the ſide C A is $=$ [z] C B. *Q. E. F.*

[w] *Prec.*
[x] *Radius's.*
[y] *Def. 7.*
[z] *XXIV.*

Problem

Problem VIII.

To divide a line [a b] *in a given Proportion.* $\frac{CA}{EA}$

A B, A C, being placed at any Ang. A, joyn B C, and from the point E, draw a Parall. [a] to it, *viz.* E D. Therefore [b] B A . D A : : C A . E A. *Q. E. F.*

[a] *Pr. I.*
[b] *LXVII.*

Problem IX.

To find a fourth proportional, [d e] *to three given,* viz. A B . B C : : A D . d e

Joyn B D, and from the point C, draw its Parall. [c] C E, joyn D E, the fourth proport. for [d] A D . B C : : A D . D E. *Q. E. F.*

[c] *Pr. I.*
[d] *LXVII. divided.*

Problem X.

To find a third Proport. [d e] *to two given,* [a b . b c]

Take A D = [e] B C, and joyn it to the point A, at any Ang. draw D B, and its Parall. [f] C E, from the point C. D E is the third Proport. for [g] A B . B C : : (B C, *i. e.*) A D . D E. *Q. E. F.*

Problem

[e] *Pr. II.*
[f] *Pr. I.*
[g] *LXVII divided.*

Problem XI.

To find a middle proportion [b c] *between two given* [a b b c.]

FRom the point B raiſe the perp. [h] B E. divide A C in the [i] middle, D. Cent. D, interv. D C describe a Semicirc. cutting the perp. in E. E B will be the middle proport. For (E A, E C, being joyn'd) the ang. A E C is [k] ∟ therefore [l] the perp. E B is a mid. propor. between the parts of the baſe A B C. *Q. E. F.*

h *Pr. VI.*
i *Pr. VII.*
k *LXI.*
l *LXXVI.*

Problem XII.

To divide a line given [Z] *in extreme and middle reaſon. So that* Z, A, E ∴ *and* Z × E = Aq.

FRom C, raiſe the perp. [m] C B = [n] Z. divide C B in the mid. in D. take [o] D F = [n] D H; and C G = C F. G, is the point of diviſion.

m *P. VI.*
n *P. III.*
o *Dem. II.*

For, □ B C F + C Fq + D C q [p] = [p] *CXI.*

(D Fq / D Fq *i.e.*[q] D Hq[r] =) Z q + D C q [q] *constr.*

therefore (omitting the com. D C q) remains [r] *CXIII.*

B C F + C Fq *i.e.*[q] Z A + Aq = Zq

But Z q [s] = Z A + Z E; therefore (Z A being com.) Z E is = [t] Aq, and by conſequence [u] Z, A, E, are ∴. *Q. E. F.*

s *CIV.*
t *Ax. 8.*
u *LXXXI.*

Problem XIII.

1. *To make an equilateral triang.* [a b c.]
2. *To make a triang. of lines given.* [d a b c.]

1. DRaw A B. Cent. A and B describe arches crossing in C. joyn C A, C B. therefore [w] all the sides are = *Q. E. F.*

[w] *Radius's of* = ○s.

2. Cent. A and B; interv. A D, B C, describe arches crossing in C. joyn C A, C B. therefore [w] A C = A D, and C B = B E. *Q. E. F.*

Problem XIV.

To make a Pgr. ([g h] *at an ang. given* [d]) = *to a* △ *given* [a b c.]

DRaw B H. [x] pall. A C. divide A C in the mid. [y] G. and make the ang. C G F = [z] D. draw C H pall. G F. and the diag. G H. (The △ G H C [a] =) ½ pgr. F C is [b] = ½ the △ A B C. therefore the whole pgr. F C is [c] = whole △ A B. C. *Q. E. F.*

[x] *P. I.*
[y] *P. VII.*
[z] *P. IV.*
[a] *XXX.*
[b] *LXXVIII*
[c] *LXIII.*

Problem

Problem XV.

To make a Pgr. [x] (*on a ſide,* [a b] *and ang.* [c] *given*) = *to a* △ *given* [D.]

MAke S = [d] D, the ang. [b] being *d* = *c.* on the line *b a*, fill [e] up the Pgr. *f a.* Run *e b* through *b*, and continue [e] to it, *f g.* from *b*, draw *b l*, pall. [f] *g a.* and through *b*, draw *i k*, [f] pall *f b*, and continue *e a* to *l.* X is the pgr. requir'd. For, the △ O X R = [g] Z S V. but O = [g] Z. R = [g] V. Therefore the remains [h] X = (S [i] =) D. and the ang. *a b k* is = [k] (*b* [i] =) *c.* *Q. E. F.*

[d] *Prec.*
[e] *Dem. II.*
[f] *P. I.*
[g] *XXX.*
[h] *Ax.* 8.
[i] *Conſtr.*
[k] *III.*

Problem XVI.

To make a Pgr. [e f g] (*on a ſide* [hi] *and ang.* [d] *given*) = *to a rectilinear figure given* [a b c.]

DIvide the rectilinear figure into △s, A, B, C; Then [k] make the pgr. E = A, according to the ang. D. and the ſide H I. Alſo F = C. and G = D: Theſe 3 will fall between the ſame parallels, becauſe [l] their angles are =, and by conſeq. make one Pgr. = [m] A B C.

[k] *Prec.*
[l] *VI.*
[m] *Ax.*

Problem XVII.

To frame a Square on a line given [a b.]

[a] *P. VI.* [o] *P. III.* [q] *Constr.* [r] *Radius's.* [s] *XVI.* [t] *XIII.* [u] *X.* [w] *VI.*

FRom A raise a perp. [a] A C = [o] A B, Cent. B and C interv. B A. descr. arches crossing at D, joyn D C, D B. *Q. E. F.* For, A B = [q] A C = [r] C D = [r] D B. C D is com. therefore [s] the △s A C B, C B D, are equiangled. Therefore A[q] = △ (L = [s]) D, but ∠ A C B is = [t] A B C, and D C B = [t] D B C. each [u] = ½ a ⌊ s. therefore C and B are ⌊, and the opposite sides are pall [w]. *Q. E. D.*

Problem XVIII.

To make a □ [d g, q] = *to a right lin'd figure given.*

[x] *P. XIV.* [y] *P. XI.* [z] *CXII.* *Schol.*

MAke [x] the □ B C = A. Between the sides of this □ find a mid. propor. [y] D G. The square of this is = [z] (B C, =) A. by constr. *Q. E. F.*

Problem

Problem XIX.

To make a □ [ed], = *to a right-lin'd figure* [a], *and* □ *given* [bc], *both together.*

MAke the □ EF = [a] A, and place the corner of it, G, againſt the □ BC, ſo that the ∠ EGB, may [b] be ∟, on the Subtend, EF, [a] make the □ ED, (which will be) [c] = (EF [d] =) A + BC. *Q. E. F.*

[a] *Prec.*
[b] *III.*
[c] *CXIII.*
[d] *Conſt.*

I. *Hence appears how one Square may be ſubtracted from another.*

II. *How a great many Squares given, may be reduced to one Square. For the* □ F, *is* [c] = Eq. (+ Iq. =) Dq. (+ Kq. =) Cq. (+ Lq. =) Bq. + Aq.

Problem XX.

To frame a Pentagon upon a line given [A B.]

Divide A B in the middle, C; raise the perp. C D = A B. Draw out D A to I, so that A I may be = A C. upon the base A B, make the △ A B F each side = D I. Lastly, upon these sides make the △s F G B, A H F, each side = A B. A B G F H is the Pentagon required. For,

Take F K = A B. and on the cent. A interv. A C, describe the ⊙ I C L. The line D I is = F A, and if you take away the = 's F K, I L, there b remains K A = L D. Now D C is a *tangent* c to the ⊙. Therefore d, I D. D C :: D C. D L. that is, F A. A B :: A B. A K. Wherefore e the △ A B F is :: A B K, by conseq. f the ∠ ζ = β. and χ = (ζ × n =) ε. Therefore the side B K = (B A = F K. Therefore the ∠ β = h n. viz. ζ = β = n. But the external ∠ χ = β × n, therefore χ, (or ε, or ζ × n) are each double to β. But ε, ζ × n, β, together, are k = 2 ∟, therefore β = ⅖ 2 ∟ and χ = ⅘ 2 ∟ (being double to β) therefore λ = l ⅗ 2 ∟. but λ m = G m = H.

Again, the ∠s F G B, F K B, F H A, are l =. (the side B K being prov'd = B A) therefore the ∠s θ, n, δ, α, γ, and β, are all = to one another, and by conseq. = ⅗ 2 ∟. Therefore δ ε, ζ n θ, α β γ, are each = ⅗ 2 ∟.

a. *by constr.*
b *Ax. 8.*
c *XLIX, constr.*
d *CXXII.*
e *LXXIV.*
f *Def. 45.*
g *XII.*
h *XIII.*
i *IX.*
k *X.*
l *I.*
m *XVI.*

2 ∟. as H x G have already been prov'd. So that all the 5 ang.'s, are [n] =. And the 5 sides are = by construction. Therefore, *Q. E. F.*

[n] *Ax.* 10.

Scholium.

The line F A *is cut in extreme and mean reason in the point* K; *for* F A. (A B, i. e.) F K :: F K. K A. *as appears in the precedent.*

Problem XXI.

To make a regular Hexagon, on a line given [a b.]

ON A B, [e] make an equal sided △ A B C. as also on A C, and B C, then drawing [f] out A C and B C, make C F and C E [g] = to them, and joyn G F E D. *Q. E. F.* For, the 3 ang.'s H C I, are = [h] 2 ∟, therefore G D is a [i] right line. Further, the ang. L is = [k] C, and L F, L E are = [l] C A, C B, therefore the △ L F E is = [m] C A B, and so of the other △s, and by consequence the two angles at E, shall be = 2 at A, and F E = A B, *&c.* So that the whole figure is equal sided and equiangled. *Q. E. D.*

[e] *P. XIII.*
[f] *Dem. II.*
[g] *P. III.*
[h] *XVI. with X.*
[i] *II.*
[k] *III.*
[l] *Constr.*
[m] *XVII.*

Problem XXII.

To make a Polygon (on a line given) [a b] like, *and alike placed, to a Polygon given* [g d.]

DIvide DG into △s, and on AB make the ∠ ABH [n] = CDE; and BAH [n] = DCE, therefore [o] the ∠ H is = E, and by conseq. the △ AHB is :: [p] CED, and so of the rest: Therefore [q] the whole DK is :: GD. *Q. E. F.*

n *P. IV.*
o *XVII.*
p *LXXI.*
q *LXIII.*

Problem XXIII.

To make a Polygon [c] like, *and alike placed, to a Polyg. given* [A], *and* = *to another Polyg. given* [B.]

ON the line DE, make [r] a Pgr. [f] = A, and on EL, the pgr. Z = [f] B. Take HI a mid. [r] propor. between DE, EK, and make on it the Polyg. C [u] :: A. This C is = B. For, $\frac{A}{C} = {}^{w}\left(\frac{DE}{EK}\text{ twice, i.e.}^{x}\ \frac{DE}{EK} = {}^{w}\frac{S}{Z}\text{ i.e.}^{y}\right)\frac{A}{B}$ Therefore C [z] = B. *Q. E. D.*

r *P. XVI.*
f *P. XIV.*
t *P. XI.*
u *Preced.*
w *XCI.*
x *Constr. bec. DE, HI, EK, are* ∴ ∴.
y *Ax. 10. constr.*
z *Ax. 10.*

Problem

Problem XXIV.

To find the center of an Arch (or Circle) given [a b c] (*or, to draw a* ○ *through 3 points given.*)

TAke 3 points in the arch, *viz.* A, B, C. joyn A B, B C. divide these in [c] the mid. [a] D and F, by [b] perp's, crossing in E, which is the Center. For each perp. is a [c] Diam. and by [d] conseq. the Center must be where they cross.

[a] *P. VII.*
[b] *P. VI.*
[c] *XXXVII.*
[d] *Def.* 22.

Problem XXV.

To divide an Arch [a c] *in the middle.*

HAving found the Cent. [e] D, draw the Radius A D G, and divide the ∠ D in the [f] mid. by the line D B; therefore the arch A B is [g] = B C. *Q. E. F.*

[e] *Prec.*
[f] *V.*
[g] *XCVI.*

Problem

Problem XXVI.

To draw to Tangent [a d] *from a point given* [a.]

h P. XXIV. DRaw a line from A to the Cent. [h] B, on A B; descr. a Semicirc. cutting the ○ in D; join A D, B B. A D, is the Tangent. For the ∠ A D B is [ ] [i]; therefore [k] &c. Q. E. F.

i LVI.
k XLIX.

Problem XXVII.

To cut off a Segment [a b] *from a* ○ *given, that may receive an* ∠ = *to an ang. given,* viz. A C B = E.

l Prec. DRaw a [l] Tangent F G, touching in B, make the ∠ F B A = [m] E. Therefore [n] the ∠ A C B (in the opposite Segment) is = (F B A [o] =) E. So that A D C B is the Segment required.

m P. IV.
n LX.
o Const.

Problem

Problem XXVIII.

To make a △ *in a* ○, :: *a* △ *given* [def.]

CUT[p] off, by AH a Segment receiving an ang. ACB = to the ∠ given, F: Then make BAC[q] = D, join BC, the ∠ BCA is = F. Therefore E[r] = B, and by conseq. △ABC :: [s]DEF. *Q. E. F.*

p *Prec.*
q *P. IV.*
r *LIV.*
s *XXII.*
t *LXXI.*

Problem XXIX.

To make a △ *about a* ○ = *a* △ *given* [def.]

MAke the ∠ at the cent. BGA[u] = EDH, and CGA[u] = EFI; and at the ends of the 3 Radius's A, B, C, draw[w] perps, meeting in K, L, N. *Q. E. F.* For, in the *Trapeze*, KBGA, the 4 ∠[s] are[x] = 4 ∟, (being by a diagonal divisible into 2 △s.) but B and A are[y] ∟; therefore K, G = 2 ∟. Now G = [y]EDH, therefore K = [z]EDF. So also M is = EFD, therefore[a] L = E; and by conseq. [b] the △s are like. *Q. E. F.*

u *P. IV.*
w *P. VI.*
x *X.*
y *Constr.*
z *I.*
a *XXII.*
b *LXXI.*

Problem

Problem XXX.

To inscribe a ○ in a △ given [a b c.]

[c] *P. V.* DIvide the ∠s A and C in the mid. [c] by lines crossing in D. From D, draw perps. to the sides of the △, or D E, D F, D G; cent. D, interv. D F, descr. the ○. *Q. E. F.* For, In the △s A E D, A D F, the ∠s at A are [d] *Constr.* =: Also A F D, A E D are [e] *Ax. 2.* =, being ∟ [d], and the side A D is com. therefore [f] *XXIII.* the side D E is = D F. = (for the same reason) D G. But D F is a Radius [d], therefore also [g] *XLVII.* must be D E and D G: and by conseq. [g] the sides B A, B C touch the ○, as A C does, by constr. *Q. E. F.*

Problem XXXI.

To conscribe a ○ about a △.

DRaw a Circle through 3 points, by P. XXIV.

Problem

Problem XXXII.

To inscribe a □ *in a* ○.

DIvide the Diam. AC in the middle [h], by the [i] perp. BD; joyn B, C, D, A, which is, the □ sought. For the 4 sides are [k] =, subtending [l] = ∠s (at the Cent,) between = sides [m], and the 4 ∠s ABCD are ∟ [n]; for AC is a Diam [o]. *Q. E. D.*

h *P. VII.*
i *P. VI.*
k *XVII.*
l *Ax. 2.*
m *Radius's.*
n *LVI.*
o *XXXVII*

Problem XXXIII.

To conscribe a ○ *about a given* □.

DRaw Diams. crossing perpendicularly [p] at the Cent. I, and at their ends, E, F, G, H, draw 4 [q] perps. meeting in A, B, C, D, which is the □ sought. For, ∠BEG [q] = EGD; therefore BA is [r] pall. CD, in like manner BC is pall. AD, and AD pall. EG, and by conseq. the ∠BAH is (= [s] BEI) ∟ [t], and the side AD [u] = EG = [w] FH [u] = BA, &c. thus all the sides will be prov'd =, and all the angs. ∟. *Q. E. D.*

p *P. VII, and VI.*
q *P. VI. Ax. 2. con.*
r *VI.*
s *IV.*
t *Constr.*
u *XXX.*
w *Diams.*

Problem

Problem XXXIV.

To inscribe a ○ in a given □.

x *P. VII. and VI.* y *Constr.* z *IV.* a *29.* b *XXX.*

DIvide the sides AB, BC, in the x mid. by perps. crossing at I. Cent. I, interv. IE, descr. a ○. *Q. E. F.* For, ∠ BEG is y ∟, = EGD; thus the ∠ FHD, is ∟ also: Therefore IH is (parl. a, and = b EA, = y EB) = a,b FI = (by the same process) IG = IE. But IE is a Radius y, therefore also must all the rest be, (being = to IE) by conseq. the ○ touches all the sides of the □. *Q. E. D.*

Problem XXXV.

To describe a ○ about a regular Pentagon.

c *P. V.* d *Ax. 7. Constr.* e *XII.* f *Suppos.* g *Constr.* h *XVII.*

DIvide the ∠s A and B in the mid. c by lines crossing in F. Cent. F, interv. FB, descr. a ○. *Q. E. F.* For, the ∠ FAB is = d FBA. Therefore FB = e FA, = h FC; because in the △s HG, the side AB = f BC, BF com. ∠ FBA g = FBC. Also, FB is h = FD. For, in the △s GI, BC f = BD; FC, com. ∠ FCB = FCD. (For FB is proved = FC; therefore ∠ FBC =

= [i] FCB; therefore FCB is ½ an ang. of the Pent. and by conseq. so is FCD.) Thus all the lines FC, FD, &c. will be proved = FB, which is a Radius [ɛ], therefore also are they; and conseq. the ○ passes by all the angs. of the Pentag. *Q. E. D.* [i] *XIII.*

Problem XXXVI.

To inscribe a ○ in a Pentagon.

CEnt. F. [k] interv. FM (*viz.* a perp. [l] from the cent. to the side) descr. a ○. *Q. E. F.* For, the ∠s at E are [k] =; and (perps. being [l] drawn to all the sides) to ∠ FME = FNE, and the side FE. com. [m] therefore FM is = FM: thus all the perps. will be proved = to FM, which is a Radius [n]; therefore also are they all: and by consequence the ○ touches all the sides of the Polygon. *Q. E. D.* [k] *Preced.* [l] *P. VI.* [m] *XXIV.* [n] *Constr.*

Problem XXXVII.

To inscribe a (regular) Pentagon in a ○ given.

MAke a reg. Pentag. [o] EB, whose cent. F being found, lay it upon the cent. of the ○; draw the Radius's FH, and through the angs. [o] *P. XX.* [p] *P. XXXV.*

angs. of the Pentag. joyn the points G, L, K, &c. *Q. E. F.* For, $\frac{GF}{HF}$ (Radius)[q] $= \frac{EF}{DF}$ (bec. EF [z] = DF.) therefore GH is [f] pall. to ED, and by conſeq. [t] $\frac{HF}{DF} = \frac{GH}{ED}$ = (by the ſame reaſon) $\frac{HI}{DC}$ &c. But ED = DC, therefore GH = HI, &c. Laſtly, upon account of the pall. lines, all the angs. are [u] = reſpectively, and (by conſequence) between themſelves; becauſe all the ∠s in the given Pentagon are [w] =. *Q. E. D.*

[q] *Ax.* 10.
[z] *Prov'd in P. XXXV.*
[f] *LXXVIII*
[t] *LXIX.*
[u] *IV.*
[w] *Suppoſ.*

Problem XXVIII.

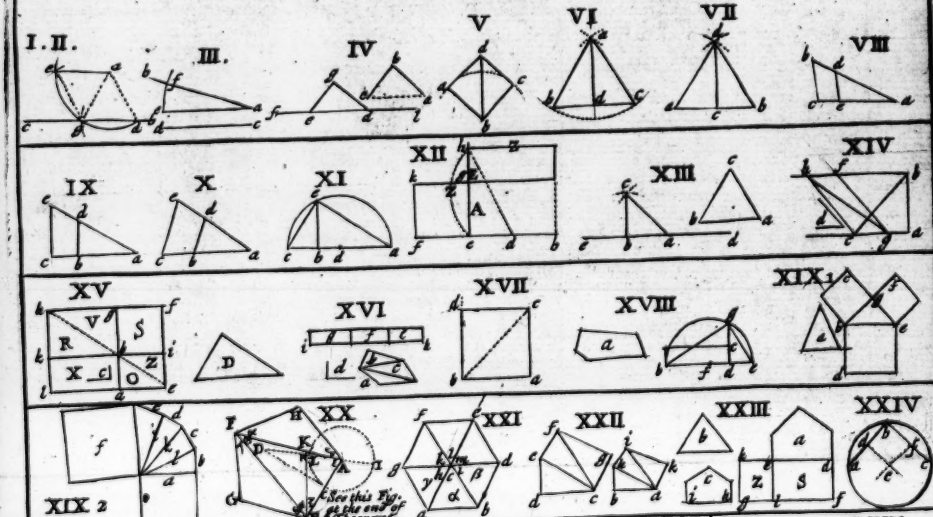
To inſcribe a Hexagon in a ○.

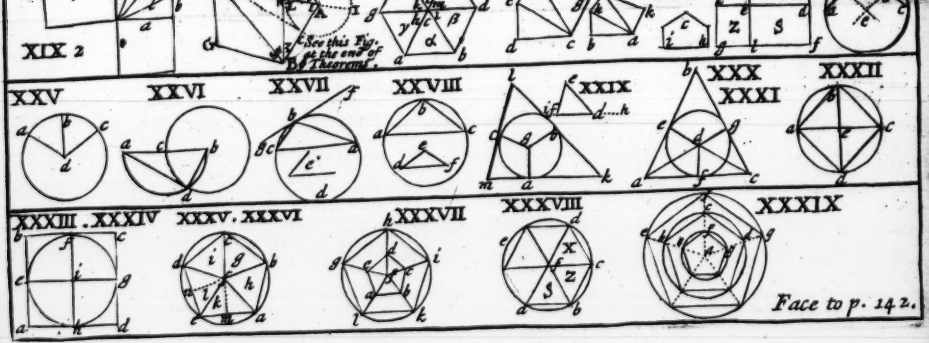
AT the interval of the Radius FA, cut off the arch AB, draw the Rad. FB, therefore [r] is an equal [x] ſided △. Then at the interv. of the Rad. FB cut off the arch B, C, &c. therefore Z is equilat. △, in like manner X. &c. therefore DA is a right [y] line, and by conſeq. a Diam. But the other half ○, DEA, will admit 3 △s = to the former; and by conſeq. the whole ○ admits a regular [z] Hexagon. *Q. E. D.*

[r] *Conſtr. Rad.*
[y] *X, & II.*
[z] *XXI.*

Problem

PROBLEMS.


I. II.
III.
IV
V
VI
VII
VIII
IX
X
XI
XII
XIII
XIV
XV
XVI
XVII
XVIII
XIX 1
XIX 2
XX
XXI
XXII
XXIII
XXIV
See this Fig. at the end of


XIX 2
See this Fig. at the end of the 1 Theorem.
XXV
XXVI
XXVII
XXVIII
XXIX
XXX
XXXI
XXXII
XXXIII. XXXIV
XXXV. XXXVI
XXXVII
XXXVIII
XXXIX
Face to p. 142.

Problem XXXIX.

A Polygon in a ○ [b c d] *being given, to inscribe a* like *Polygon* [e f g] *in a* ○ *of any different size.*

UPon the center of the given ○, A, describe the other ○, E F G; draw the Radius's through the ∠s of the Polyg. given, B, C, D, &c. joyn the ends of these Radius's with the right lines E F, F G, &c. The Polyg. E F G will be like the Polyg. B C D. For the △ A B C is :: ° A E F.

Because the 3 ∠s at A, B, and C, together, are ᵖ= (2 [ᵖ=) A, E and F together; therefore taking away from these = sums, the common A, there remains B C = ᵍ E F. But, B = ʳ C, and E ʳ= F, therefore B and E, (being like parts, *i. e.* halfs, of = sums) are = ˢ; therefore the line E F is ᵗ pall. to B C.

In like manner, all the other △'s will be proved *like*; therefore the whole ᵘ Polygons are like. *Q. E. F.*

° *LXX.*
ᵖ *X.*
ᵍ *Ax.* 10
ʳ *XIII.*
ˢ *LXII.*
ᵗ *VI.*
ᵘ *LXVII.*

End of the Problems.

If the Reader consider the Difficulty of Printing Books of this nature without faults, and the greater trouble he would be at if they were not faithfully Corrected; 'tis to be hop'd he will pardon this large Catalogue of

ERRATA.

IN the *Advertisement*, *line* 3. *read* Principles of. *Def.* 6. *l.* 8. *r.* Angles. *Def.* 30. *l.* 2. *r.* [ac, bc.] *marg. add* Fig. 16. *Def.* 31. *marg. dele* Fig. 16. *Def.* 35, 2, *Marg. r.* Fig. 20. *Def.* 41. *Marg. r.* Fig. 37. *Def.* 43. *marg. r.* Fig. 38. *Def.* 51. *l.* 3. *after* angle, *r.* B A E. A, E, being the points where, &c. *Ax.* 3. *l.* 3. *for* [b] *r.* [b d.] *at the end*, *r.* [b c.] *marg. add* Fig. 6. *Ax.* 12. *l.* 4. *for* A to B. *r.* C to A. *explic. of the Notes. l. last. r.* Hyp. Sup. *add* Q. E. D. which was to be demonstrated. Q. E. A. which is absurd. Q. E. F. which was to be done. *Theor.* I. *l.* 1. *r.* Perp. *Theor.* II. *l.* 4. *r.* ABC. ABE. *Theor.* III. *l.* 3. *for* ^{ff}. *r.* ^{ee}. *l.* 4. *r.* C is ^{f} = to D. *marg. r.* ^{f} *Ax.* 8. *Theor.* 4. *dele* ^{k}. *Theor.* 16. *l.* 3. *r.* BC =. *Theor.* 24. *title l.* 2. *after* are =. *r.* [abc. def.] *p.* 12. *l. last. r.* ^{m} be the. *p.* 13. *l.* 1. *after* therefore, *r.* in the △ B γ C. *Theor.* 41. *title l.* 1. *for* c d. *r.* a d. *Th.* 51. *l. last. for* F. *r.* B. *p.* 48. *l.* 4. *for* D G H. *r.* C D G. *Th.* 55. *l.* 3. *after* B = A. *r.* by 54. *Th.* 56. *l.* 4. *for* ⌊ *r.* 2 ⌊. *Th.* 59. *title l.* 3. *after* Circumference. *r.* [a b c.] *Th.* 60. *marg. for* ^{h} L. *r.* ^{h} I. *and for* ^{i} I. *r.* ^{i} 62. *Th.* 63. *l. last. for* C 6. *r.* D 6. *Th.* 69. *l. last. for* F + G. G. *r.* F + E. (E. i.e.) G. *Th.* 72. *marg. after* Hyp. *r.* (*viz.* D + C. B + A :: δζ. δε. and D + C. E :: δ. ζ ε ζ) *l.* 3. *r.* C^{d} or. *Th.* 74. *marg. r.* ^{r} 70. *Th.* 75. *l.* 2. *for* (δ ζ ^{f} or) A, δε. *r.* (δε ^{r} or) A δζ. *marg. r.* ^{w} 4. *Th.* 77. *l.* 1. *for* IH. *r.* FH. *Th.* 80. *l.* 1. *for* C D. *r.* C B D. *marg. r.* ^{m}. 78. alterned. *Th:* 81. *marg. for* ^{p} 72. *r.* ^{p}. *Ax.* 7. & 30. *Th.* 84. *title l.* 1. *after* [s, o] *r.* (i. e. α S β O.) *Th.* 89. *marg. r.* ^{k} Def. 45.

p. 72

p. 72. *l. last but* 4. *for* F I. *r.* H I. *last but* 2. *r.* H F I. *Th.* 92. *l.* 8. *r.* ○ it self; which we conceive to be only a regular Polygon of infinite sides, and by, *&c. marg. after* Prob. *r.* 39. *Th.* 93. *title. r.* 4 ∴ S. 3. *l.* 2. *r.* each side against the other, so drawn but makes 2 Angles ∴ *l.* 4. *after* outward, *r.* at the sides 9. 4. N. *r.* Angles, at the sides. *Th.* 94. *l. last but* 4. *dele* k. *Th.* 96. *l. last. r.* Arch B D. *Th.* 97. *l.* 3. *r.* N B. *l.* 7. *r.* O H B. *l. last but* 2. *r.* G Z B. *Th.* 102. *title l.* 2. *r.* equal in compals. *P.* 80. *l.* 8. *r.* A E or A. *l.* 12. *add*, Note also that B stands for Summa, or Summa, *i. e.* the whole line; and A, E, most commonly for the parts of it, A the greater, and E the less. *Th.* 112. *l.* 1. *for* T E. *r.* I F. *marg. r.* 4. Def. 45. *Th.* 115. *title l.* 1. *after* Triangles, *r.* [A B C.] *Th.* 122. *after* therefore, *r.* the ∟. *p.* 94. *l.* 2. *r.* $\left\{\begin{matrix} \text{D C} \\ \text{D A.} \end{matrix}\right.$ *p.* 96. *l. last. r.* Def. 55. *Th.* 133. *l.* 4, 8, *&c.* for 60. *r.* 50. *Th.* 142, *l.* 4. *for* B E. *r.* A E. *l. last but one, for* B A C. *r.* D A C. *Th.* 144. *marg. r.* k Def. 78. *and* P Def. 45. with Ax. 10. *Th.* 146 *l.* 2. *for* K F. D A. *r.* D C. P A. *p.* 105. *l.* 3. *after* Parallels. *r.* H A. L C. *l.* 4. *after* S Z. *r.* viz. P D. R G. *l.* 5. *for* = prov'd. *r.* prov'd ≠. *l. last but one. r.* G C. (= E C.) *l. last. r.* A H (= S.) . X. *p.* 107. *l. last r.* which $\frac{S}{X}$ thrice $= \frac{AE}{ED}$ *thrice*; *by* 73 ª Q. E. D. For $\frac{S}{X}$ once is $= \frac{AE}{ED}$ once; as hath been prvo'd before. *p.* 109. *l.* 2. *dele* be. *Th.* 156. *l.* 3. *r.* are in a. *p.* 112. *l. last. for* Q. E. D. *r.* Q. E. A. *Th.* 158. *title. l.* 3. *for* E Z. *r.* E F. *p.* 115. *l. last but one. for* side. *r.* sphere. *Th.* 164. *l. last but* 2. *r.* But the Sph. *p.* 117. *l.* 8. for 4 □. *r.* 3 □. *marg. dele* Fig. 164. *p.* 120. *l.* 2. *r.* propose. *Prob.* 6. *l* 8. *r.* B C joyn. *l.* 9. *for* time. *r.* line. *p.* 7. *l. last but one. for* D. *r.*

C.

ERRATA.

C. *p.* 9. *l. laſt. r.* for AB. *p.* 11 *title line* 1. *for* bc. *r.* be. *Text. l.* 2. *after* interv. *r.* dc. *or* d2. *p.* 12. *l. laſt but* 3. *r. it thus.* Zq = (BCF + CFq *i.e.* qZA + Aq *p.* 13. *l.* 4. *for* BC, *r.* BE. *p.* 15. *l.* 1. *r.* the ∠ b, being = dc. *p.* 17. *l.* 6. *for* = △ (*L. r.* = (□. *p.* 18. *title. for* □. *r.* □. *after* given. *r.* A. *p.* 19. *l.* 4. *for* EF. *r.* EB. *p.* 20. *for* ζ × *n. r.* ζ + *n in ſeveral places. pag.* 131. *l.* 1. *r.* H and G. *pr.* 23. *l. laſt but one. for* EK twice. *r.* IH twice. *p.* 26. *l.* 3. *r.* AE, BD, AD. *p.* 28. *l.* 4. *after* F. *r.* and BAC = EDF. (conſter) *marg. r.* ˢ Sup. with LIV. *p.* 29. *title. for* = *r.* :: *p.* 30. *l.* 3. *for* or, *r.* as. *p.* 32. *l. laſt. for* AC. *r.* BD. *p.* 33. *title. for* ○ & □. *r.* □ & ○. *p.* 36. *l.* 1. *dele* ᵇ. *l.* 4. *r.* the ∠. *l.* 6. *r.* FN *is. Pr.* 37. *l.* 2. after found. *r.* (as in Prob. 35.) *l.* 3. *for* and. *r.* &c. *marg. r.* ᶠ 68. *p.* 140. *l.* 12. *r.* Pr. 38. *l.* 3. *for* ᶠ. *r.* S. *marg. r.* ᵇ. Pr. 21.

FINIS.

INDEX

With reference to *EUCLID.*

ANGLES.

	Theor.	Euclid.
Angles. Above a line are = 2 ∟.	1	1, 13
At croſſing lines, the oppoſite are =.	3	1, 15
At parall. lines, are = to their Verticals.	4	1, 29
——— the 2 oppoſite are = 2 ∟.	5	
In a △. The external ∠ is = to the 2 internal oppoſite.	9	1, 32
The 3 ∠s together are = 2 ∟.	10	
That ∠ is ∟, the Q. of whoſe ſubtend. is = to both the Q's of the other ſides.	116	1, 48
In a ſolid ∠. Two plane ∠'s are greater than the third.	142	11, 20
A ſolid ang. conſiſts of leſs than 4 ∟.	143	11, 21
In a ○. The ∠ of the Tangent and Radius, is ∟.	49	
——— of the circumf. and Radius, is ⊏ than any acute ∠.	84	
All ∠s in the ſame ſegment are =.	54	3, 21

∠ at the Cent. is double to that at the circumf.	55	3, 20
The ∠ in a semicirc. is ∟.	56	3, 31
——In a segment ⊏ is acute. ⌝ obtuse.	57	
= angs = arches.	58	3, 26
The ∠s are proport. to the arches.	96	
The ∠s of the tangent and chord is = to the ∠ in the oppoſite ſegment.	60	3, 32
The external ∠s of every Polyg. are = 2 ∟.	93	
Problems. To make an ∠ = to an ∠ given.	P. 4	1, 32
To divide an ∠ in the mid.	5	1, 9

Bodies, or Solids.

ALL :: bodies, are in tripl. reaſon of their homol. ſides.	161	
Ppps. The oppoſite planes are :: and =.	144	11, 24
They are divided equally by a diagonal plane	145	11, 26
Of the ſame height are as their baſes.	148	11, 32
Which have the ſame, or = baſes, are =.	146 / 147	11, 19, 30 / 31
=, have their baſes and height reciprocal.	149	
——————and convertedly.	150	11, 34
::, are in tripl. reaſon of their homol. ſides.	151	11, 33
—are as the cubes of their homol. ſides.	152	
Prz. and Cyls. have the ſame properties.	153 / 154	11, 40 / 12, 11, &c.
△ Piz. is divided into 3 = Pyram's.	158	12, 7

Pyram. is divided :: by a plane pall. to the base. 155
Of = height are divided into = segms. by a plane pall. to their bases. 156
Are =, which have = bases and heights. 157 12, 6
They have the same proprieties with Ppps. 159 12, 89
Cones have the same propr's with Pyrams. 160 12, 10
Spheres are in tripl. reason of their Diams. 164 12, 18,
Are = to a Cone whose axis is the Rad. and its base = curv surface of the Sph. 162
Are ⊏ than all solid figures of = surface. 163
Polygons regular are but 5. 165

CIRCLES.

THe cent. is from whence more than 2 = lines can be drawn to the Circumference. 38 3. 9
○s touch (within, and without) but in one point. 45, 3, 13
And the com. Diam. falls on the point of touching. 43, 44 3, 11, 12
○s cut themselves but in 2 points. 46 3, 10
They are in a dupl. reason of their Diams. 92 12, 2
Arches and ⌊s subtended in the same reason. 96 6, 33
○ = ⌊ △, of which one side the Rad. the other the compass of the ○. 99

○ contains more ſpace than any figure of = compaſs.	102	
The ſquaring of the Lunes *Hippocr.*	119	
[*Problems.*] to find the Cent.	24	3, 1
To divide an Arch in mid.	25	3, 30
To cut off a Segm. for any Ang.	27	3, 34
To inſcr. a ○ in a △.	30	4, 4
To conſcr. a ○ about a △.	31	4, 5
To conſcr. a ○ about a □.	33	4, 7
To inſcr. a ○ in a □.	34	4, 8
To conſcr a ○ about a Pentag.	35	4, 14
To inſcr. a ○ in a Pent.	30	4, 13

LINES.

THe ſhorteſt from a point to a Line is a perp.	25	
A right line, which	2	7, 14
	131	11, 3
Palls which.	6.7	1, 27, 28
	7.8	6, 2
To a Third are palls to one another.	8	1,30,11,9
Are in the ſame Plane.	180	
Dig ms in a □ cut themſelves mid.	31	
In a ○ The Diam. is the greateſt.	52	3, 15
——————is perp. to the mid. of a Chord, and converſedly	35, 36	3, 3
The line which does ſo is the Diam.	37	
Lines out of the cent. do not cut in the mid.	39	3, 4
Of Lines drawn from a point *in*, or *out of* a ○ to the Circumf. the greateſt paſſes the cent. *&c.*	40, 41	3, 7

If

If from a point without the ○, to the convex circumf. the line would pass the cent.	48	3, 8
Lines = distant from the cent. are =.	51	3, 14
Lines in a ○ cut themselves proport.	120	3, 35
A perp. to the Diam. from the circumf. is a mid. proport. between the parts of it.	120	
Two lines from a point without to the concave circumf. are as their outward parts.	123	
A whole secant is to the Diam. as the part of the Diam. cut off by a perp. from the end of the secant, is to the inner part of the secant.	124	
A chord dividing an ∠ in a segm. is to one side of that ∠ as the other side, is to the part of the chord within the segm.	125	
Tangents make a ∟ to the end of the Radius.	49	3, 18
A line that does so, is without the ○.	47	3, 16
Tang. is a mid. proport. between the whole secant, and its outward part.	122	3, 36
Two tangs. drawn from the same point, are =.	50	
In a plane Crossing lines, are in the same Pall. plane.	129	11, 2
	130	11, 5
No right line can be in two planes.	128	11, 1
A perp. to 2 crossing lines is perp. to their planes.	133	11, 4
If a line be perp. to a plane, all its palls. are so.	134	11, 8
A perp. to a plane, is perp. to its pall. plane, and convertedly.	137	11, 14

If

If the 3 lines have the same perp. they have the same plane.	134	11, 5
But one perp. can be rais'd from one point.	136	11, 13
Two pair of meeting lines, pall. in different planes, contain = ∠s.	140	11, 10
Pall. lines are cut proport. by pall. planes.	141	11, 17
[*Problems*] To draw a pall,	*P.* 1	1, 32
To draw a line = line given.	2	1, 2
To cut off a part = line given.	3	1, 3
To raise or let fall a perp.	6	1, 11, 12
To divide { midd.	7	1, 10
a line in { a proport. given.	8	
To divide a line in extreme and mid. reason.	12	2, 11, 6, 30
To draw a tangent from a point.	26	3, 17

Parallelograms, and figures of four sides.

A *Pgr.* is divided in the mid. by the Diam. and its opposite sides, and ∠s, are =.	30	1, 34
Two Diagons cut themselves in the mid.	31	
A line passing the mid. of the diag cuts the pgr. in two.	32	
The Complements are =.	33	1, 43
Pgrs. are = which have = bases and heights.	34	1, 35
Proport. of the same height, are as their bases.	80	6, 1

=

= angled, have ſides recipr. proport.	81	6, 14
= angled, have a reaſon compounded of their ſides.	82	6, 23
Are = , which have ſides recipr. proport.	84	
::, have a dupl. reaſ. of their homol. ſides.	86	
::, receive the ſame diam.	89	6, 24
□s are :: □s.	85	
A 4 four ſided fig. in a ○, has oppoſ. ∠s = 2 ∟.	53	3, 22
——— has the □ of the diags = to both the □ of the oppoſ. ſides.	127	
□ about, is double to □ in a ○.	126	
See Rectangs.		
[*Problems.*] To make a Pgr. (at an ∠ given, = △ given.	p.14	I. 42
(On a ſide and ∠ given) = △ given.	15	I, 44
(On a ſide and ∠ given) = any fig. given.	16	I, 45
To make a □ on a line given.	17	I, 46
To make a ▭ = any. fig given.	18	2, 14
To make a □ = to a fig. and □ together.	19	
To inſc. a □ in a ○.	32	4, 6

PLANES.

THe Interſection of two planes is a right line.	131	11, 3
The interſcs of 2 pall. planes by a third plane, are pall.	132	11, 19

Planes are paſſ. which have the ſame perp.	138	11, 14
——which have two pair of paſſ. lines reciprocally croſſing.	139	11, 15

Polygons; Figs. of ſeveral ſides.

:: Figs. may be divided into an = number of :: △s.	90	6, 20
——are in a dupl. reaſ. of their homol. ſides.	91	
——are proport. if their baſes be ſo.	95	6, 22
With a ◯. A Polyg. about a ◯, is = to a ⌊ △, whoſe baſe is the compaſs, and perp. the rad.	97	
Every reg. Polyg. is = ⌊ △, whoſe baſe is the compaſs, and perp. = to perp. rad. of the Polyg.	98	
A Polyg. has all its external ∠s = 2⌊.	93	
3 only reg. fig's fill a ſpace.	94	
[*Problems.*] To make a reg. Pentagon on a line given.	20	
To make a reg. Hexagon.	21	
To make a Polyg. :: a Polyg. given.	22	6, 18
And = to another Polyg.	23	6, 25
To inſc. a Pentag. in a ◯.	37	4, 11
To inſcr. a Hexag. in a ◯.	38	4, 15
To inſc. a Polyg. (reg. or irreg.) in a ◯ :: a Polyg. in a ◯ given.	39	

Proportionals.

Alterned, Inverted, Compounded, Divided.	61 to 66	
Like parts are as their wholes.	63	
[Problems.] To find a fourth proport.	P. 9	6, 12
——— a third.	10	6, 11
——— a mid.	11	6, 13
To divide a line in extreme and mid. reas.	12	2, 11 and 6, 30
□ of the extremes = □ of the 2 mid.	112	
□ of the extremes = □ of the mid.	Schol.	

Rectangles, or the power of lines.

B Z = BA + BE.	103	
B Z = CA + DA + CE + DE.	Schol.	2, 1
Z A = A A + Æ.	104	
Z q = Aq + Eq + 2 Æ.	105	2, 3
Z q + Eq = 2 Z E + Aq.	106	2, 4
Q Z + E = 4 Z E + Aq.	107	
Q ½ Z = A + E × B + Eq.	108	
Z q + Bq = Aq + 2 Eq.	109	
Z q + Eq = ½ Q ½ A + 2 Q ½ A + E.	110	
Q ½ A + E = Æ + Q ½ A + Eq.	111	
□ of the extremes is = □ of mid.	112 Schol.	

Tri-

Triangles.

In a △ 2 sides are ⊏ than the third.	11	I, 20
= ∠s, = subtend's, and convertedly.	12, 13	
⊏ ∠ gives the ⊏ subtend. and convert.	14, 15	I, 18, 19
A line cutting in two the base (of an = legd. △) is perp. to it.	18	
In △s = sides, = ∠s.	16	I, 8
Are = angl. { which have 2 ∠s =, &c. / whole sides are proport.	22 / 73	
Two sides and one ∠ (between, and not between) =, all =.	17, 24	I, 4
Two sides =, and one ∠ ⊏, its subtend. is ⊏; and convert.	19, 20	I, 24, 25
One side, and 2 ∠s =, all =.	21, 23	I, 26
On the same, or = bases, have the same height.	27, 29	I, 39, 40
Proport. Of the same height, are as their bases.	78	6, 1
Their sides are cut proport. by a pall. to the base.	67	6, 2
And the part upwards is :: the whole △.	70	
The base is to the pall. as the sides to the parts above.	69	
△s *are like*, whose { Ang's are =.	71	
Sides are proport.	72	
Sides about one = ∠ are proport.	74	
2 sides are proport. &c.	75	

△s

{ △s with one ∠ =, have sides, about this, recipr. proport.	79	6, 15
And have a reaſ. compounded of their ſides.	87	
:: △s { Have a dupl. reaſ. of their homol. ſides.	88	6, 19
:: △s { Are, as their baſes.	79	
Sides of a △ are proport. to the parts of the baſe, a line cutting in two the oppoſite ∠.	77	6, 3
A ∟ △ is divided into parts :: the whole, by a perp. to the baſe.	76	6, 8
And the perp. is a mid. proport. between the parts of the baſe.	*id.*	
The Qs of the ſubtend, = 2 Qs of the other ſides.	113	1, 47
Any fig. upon the ſubtend, = 2 :: figs. on the other ſides.	117	6, 31
Semicirc. = 2 Semicircs.	118	
In ∠ △. Q ſubtend ⌐ than the other 2.	114	2, 13
In obtuſe △, Q ſubtend ¬, *&c.*	115	2, 12
A △ in a ſegm. is ⌐ ½ ſegm.	100, 101	
Every △ is in the ſame plane.	129	
[*Problems.*] To make an = lat. △, or a △ of lines given.	*p.*13	1, 1 & 22
To make a △ in a ○ :: a △ given.	28	4, 2
To make a △ about a ○ = △ given.	29	4, 3

F I N I S.